Math 235 - Dr. Miller - HW #4, Spring 2024 - **Due by 4pm on Friday, Feb. 9, 2024**

1. I forgot to give you a computational problem about the Division Algorithm, so here it is now:

   Find the quotient q and remainder r promised by the Division Algorithm for each setting below. Then separately rewrite the results in equality-form $a = bq + r$.

   (a) Dividend $a = 73$ and divisor $b = 6$ (The divisor is the number you're dividing BY.)
   (b) Dividend $a = 6$ and divisor $b = 73$
   (c) Dividend $a = -6$ and divisor $b = 73$
   (d) Dividend $a = -73$ and divisor $b = 6$
   (e) Dividend $a = 15$ and divisor $b = 3$
   (f) Dividend $a = -15$ and divisor $b = 15$
   (g) Dividend $a = -20$ and divisor $b = 3$

2. Choose either proof by contrapositive (ctp) or proof by contradiction (⨳) to prove each proposition below. Be rigorous!

   (a) Prop. - There is no smallest odd integer.

   (b) Prop. - Let a be a real number. If $(a + 2)(a - 3) < 0$, then $a < 3$.

   (c) Prop. - $\sqrt[3]{23}$ is irrational.

   (d) Prop. - $\sqrt{45}$ is irrational.

   (e) Prop. - Let $x, y \in \mathbf{Z}$. If $x \neq y$, then the square of their sum cannot equal 4 times their product.

   (f) Prop. - Let $c \in \mathbf{R}$. If the graphs of $x^2 + y^2 = 1$ and $y = x + c$ intersect, then $c < 5$.

3. Write a 2-part (2-direction) proof for the following biconditional statements. Remember that you can use a different style on each direction.

   (a) Let $b \in \mathbf{Z}^+$. Then $b|(b - 3)$ if and only if $b|3$.

   (b) Let $m, n \in \mathbf{Z}$. The expression $5(m + n) + 8$ is odd if and only if m and n have different parity.