1. Write a chain-style proof of the following: *Let $a, b, c \in \mathbf{Z}$ with $c \neq 0$. Then $ac|bc$ if and only if $a|b$.*

2. Rigorously prove the following quantified statements either directly or constructively.

   (a) Proposition: Every number of the form $3(2 - r)$ where $r \in \mathbf{Q}$ is also of the form $\frac{5t}{2}$ where $t \in \mathbf{Q}$.

   (b) Prop.: For every point on the graph of $x^2 + y^2 = 9$, the sum of its coordinates is less than 8.

   (Be careful not to tangle the function variables x and y with variables to use as proof variables.)

   (c) Proposition: The function $f(x) = \dfrac{x^3 + 1}{x + 1} + x$ is even (when $x \neq \pm 1$).

   (We'll review Monday, but our text defines *odd function* on p.52; it is a "for all" definition. *Even function* is similar and can be found all over the web or in your pre-calculus/calculus texts.)

   (d) Proposition: There exists a prime number x for which $x+2$, $x+3$, $x+4$, and $x+5$ are composite.

   (e) Proposition: There is a set of 10 consecutive integers containing 2 separate pairs of twin primes.

   (Twin primes are prime numbers that differ by 2.)

   (f) Proposition: There exists a rational number s where $30 < s^2 < 35$ and $180 < s^3 < 190$.

3. Prove via an ALTERNATIVE style other than direct/constructive: *Any pair of real numbers whose sum exceeds 300 must involve at least one number over 150.*

4. Rigorously prove the following. Be careful with the multiple NTS lines that should occur in proving these mixed-logic statements.

   (a) Proposition: For each $n \in \mathbf{Z}^+$, there exists an $M \in \mathbf{Z}^+$ such that $\dfrac{1}{M - 1} < \dfrac{1}{3n}$.

   (b) Proposition: There exist $a, b \in \mathbf{Q}$ such that, for all $x \in \mathbf{R}$, $\dfrac{ax}{2} + \dfrac{b - 6}{7} = x$.