1. Rigorously prove the following. Be careful with the multiple NTS lines that should occur in proving these mixed-logic statements. Also realize that sometimes, our "candidate" will be a formula found by scratchwork, and not a concrete number.

   (a) Proposition: Let $x \in \mathbf{R}$. Then $x = 0$ if and only if, for all $y \in \mathbf{R}$, $xy = x$.

   (b) Proposition: For every integer n, there is an even integer $k \in (n, \infty)$.

   (c) Proposition: There exists $e \in \mathbf{R}$ such that $xe - x - e + 2 = x$ for all $x \in \mathbf{R}$.

   (d) Proposition: For all $x \in \mathbf{R} \setminus \{1\}$, there exists $y \in \mathbf{R}$ such that $xy - x - y = 0$.

2. Rigorously prove the following uniqueness results.

   (a) Proposition: The antiderivative $F(x)$ to $f(x) = 3x^2 - \cos(x)$ such that $F(0) = 3$ is unique.

   (Be careful: getting only one answer from your process to *find* $F(x)$ is not the same as *proving* that $F(x)$ is unique.)

   (b) Proposition: The positive solution to $ax^3 + bx + c = 0$ is unique when $a, b, c \in \mathbf{R}^+$.

   (c) Proposition: Suppose there exists a real number a such that $ax + 2^x = 0$ for all $x \in \mathbf{R}$. Then a is unique.

3. Complete these problems from the Exercises 2.1 and 2.2 sets in our book:

   (a) p.92: #4 parts b,d,f,h,j only
   (b) p.92: #5 parts b,d,f,h,j,l only
   (c) p.93: #15 parts b,d,f,h only
   (d) p.93: #16 parts b,d only
   (e) p.93: #17 parts b,d,f,h,j,l only
   (f) p.101: #1 parts b,d,f,h,j only
   (g) p.101: #2 parts b,d,f,h,j only
   (h) p.102: #13 part a only

4. Rigorously prove:

   (a) Proposition: Let $X = \{5, 6\}$ and $Y = [5, 6]$. Then $X \subseteq Y$ but $Y \not\subseteq X$.

   (I expect you to use a formal definition of intervals here: $a \in [c, d]$ if and only if $c \leq a \leq d$.)

   (b) Proposition: Let $S = \{6a - 15b \mid a, b \in \mathbf{Z}\}$. Then $S \subseteq 3\mathbf{Z}$.