Math 235 - Dr. Miller - HW #7, Spring 2024 - **Due by 4pm on Friday, Mar. 8, 2024**

1. Prove rigorously: *Let* $T = \{x \in \mathbf{R} \mid x^2 \leq x\}$. *Then* $T = [0, 1]$.

2. Prove via chain-style proof. At the end of each line, also label in parentheses the specific definition or just "rules of logic" that you used in that line of the proof; we did this in class.

   (a) Let A, B, C be sets. Then $A \cap (B \setminus C) = (A \setminus C) \cap B$.

   (b) Let A, B, C be sets. Then $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

   (c) For all sets A, B, $(A \cap B^c)^c = A^c \cup B$.

   Note the slight variation in wording here; does that affect the proof?

3. Proofs that something equals the empty set often go better by contradiction than by two-part ($\subseteq$, $\supseteq$) equality proof or chain-style proof, since statements like "Let $x \in \emptyset$" or "$x \in \emptyset$ iff..." don't make sense right from the start!

   Rigorously prove this proposition, by contradiction: *For all sets* A, $A \cap \emptyset = \emptyset$.

4. Prove by any meaningful method:

   (a) Let A, B be sets. Then $A^c \subseteq B$ if and only if $A \setminus B = B^c$.

   (b) For all sets A, B, C, D, if $C \subseteq A$ and $D \subseteq B$, then $C \cap D \subseteq A \cup B$.

   (c) Let A, B, C, and D be sets with A and B disjoint. If $A \cup B \subseteq C \cup D$ and $C \subseteq A$, then $B \subseteq D$.