Math 235 - Dr. Miller - HW #10, Spring 2024 - **Due by 4pm on Friday, Apr. 12, 2024**

Prove the following via mathematical induction:

1. $\prod_{i=1}^{n}(2i-1) = \frac{(2n)!}{n!2^n}$ for all $n \in \mathbf{Z}^+$

2. For all integers $n \geq 2$, the sum $\sum_{i=2}^{n+1} i \cdot 2^i = n \cdot 2^{n+2}$.

3. For all integers $n \geq 0$, the product $\prod_{i=0}^{n}\left(\frac{1}{3i+1} \cdot \frac{1}{3i+2} \cdot \frac{1}{3i+3}\right) = \frac{1}{(3(n+1))!}$

4. For every integer $n \geq 2$, we have $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{(n-1)n} = 1 - \frac{1}{n}$.

   (You should NOT rewrite this formula to use Σ or Π, but you SHOULD review from Discrete Math how we interpet and work with a summation that has been written in expanded form, as this one has.)

5. $1^2 - 2^2 + 3^2 - \cdots + (-1)^{n+1}n^2 = \frac{(-1)^{n+1}n(n+1)}{2}$ for all $n \in \mathbf{Z}^+$

6. $7 \mid (3^{2n} - 2^n)$ for all $n \in \mathbf{Z}^+ \cup \{0\}$

7. $9 \mid (4^{3n} - 1)$ for all integers $n \geq$ _?_, where you determine the correct "base case"

8. $15 \mid (14^{2n-1} + 1)$ for all integers $n \geq$ _?_, where you determine the correct "base case"

9. $6n + 8 \leq 7n$ for all integers $n \geq$ _?_, where you determine the correct "base case"

10. $3^n + 100 \leq 4^n$ for all integers $n \geq$ _?_, where you determine the correct "base case"