Math 235 - Dr. Miller - **REVISED** Topics List for Exam #1, Spring 2024 - **Friday, Feb. 16, 2024**

Prepare by studying the topics listed below, in conjunction with your notes, text/readings, any handouts, Short Practice, and graded and ungraded HW problems. You will be more successful if you strive to master the concepts in general, rather than simply memorizing specific examples that we have already done. Studying together is a plus.

Non-Proof Tasks: Discrete Math review tasks like HW #1 will be very limited on our exam (less than 10%).

1. Be familiar with the set names $\mathbf{Z}$, $\mathbf{Z}^+$, $\mathbf{Z}^-$, similar variations for $\mathbf{Q}$, $\mathbf{R}$.
2. State hypothesis or conclusion in a conditional statement that's written using: if-then, trailing if, only if (including "split"), necessary, sufficient, implies, when/whenever.
   (a) Your response should be a stand-alone sentence, and it CAN'T keep any of the conditional words, such as if/when, then, only, necessary, or sufficient.
3. Write converse, inverse, contrapositive, including with a forced choice of phrasing from above.
4. Negate statements that contain: conditional, and, or, at least/most, $>, \geq, <, \leq, =$, 3-way inequalities.
5. For use in proofs, know the possible remainders when we divide by a specific small positive number; remember that $a \not| x$ creates cases for x's remainder.
6. State the "if" versus "only if" direction of a biconditional statement in unchanged order OR in if-then form (my choice). If asked, label with the correct choice of $\Leftarrow$ or $\Rightarrow$.
7. State FORMAL defns: rational, even, odd, Division Algorithm, divides, FTA,
   (a) Formal definitions use variables, VERBAL quantifiers (not $\forall$ or $\exists$ symbols) and buffer words. Don't use commas in place of buffer words.
   (b) Formal definitions often have universal hypotheses too. You must state those.
   (c) Math definitions aren't just a description of a word's meaning: rather, they are PRECISE phrasings that perfectly control the associated logic. MEMORIZE clear, complete definitions to earn full credit.
8. Know the different synonyms within divisibility: divides, factor, multiple, divisor, divisible by.
9. Write the logical equivalences governing proof by: cases, "or conclusion"-style, proof by ctp, 2-part. Truth tables WILL not appear on this exam.
10. Given a dividend and divisor (know the difference), find the quotient and remainder guaranteed by the Division Algorithm. Prepare for some negative numbers.

Proof Tasks: Using direct, cases (including "wlog"), "or conclusion," contrapositive, contradiction, 2-part

1. I may require you to use MY choice of proof style or leave that choice to you. Practice so that when the choice is yours, you can decide rather quickly. (The advice in Section 1.7 of our text can be helpful.)
2. In set-up tasks, I ask you only to launch the proof (meaning to write universal hypothesis, what we assume, and what we "NTS") and to give one meaningful sentence that could come next.
   (a) Set-up tasks almost always require MY choice of proof type.
   (b) Be clear about the separate directions ("if" versus "only if") in a biconditional statement: I may ask you to set-up or to prove just ONE direction (possibly my choice, or possibly yours).
      i. Remember that I identify these verbally as the "if" direction versus the "only if" direction.
      ii. Know which direction is which in a verbal setting, and also give the correct $\Rightarrow$ versus $\Leftarrow$ label in your launch, to go with the direction I specify.
   (c) **ADDED after 2-5-2024 class discussion**: Given the launch of a "proof," name what kind of proof the writer has attempted - or if their launch doesn't correctly match any of our known types.
3. Proofs can/might be about the following concepts:
   (a) Formal definitions of parity, rationality, remainders, divides
   (b) Algebraic concepts: equation/inequality solving, equations of lines or circles, distance formula, , etc.

(continued on back)

4. Remember these proof components:
   (a) You MUST write your explicit assumptions at the outset, including universal hypotheses.
   (b) You MUST write a formal NTS claim; a "meaning" add-on is helpful but not required.
   (c) You MUST be clear that you have proved what was asked, via the "exit move" sentence.
   (d) Write in SENTENCES. Proofs are never just a string of equalities or algebraic expressions. Such things must appear in context with words telling what they represent/what you're doing with them.
5. I give PARTIAL CREDIT if you have the framework correct for your proof, even if you can't do anything past the NTS. So put on the "Thus" and "exit move" ending sentences even if you're stuck on the middle.

General Advice:

1. There will be 3-4 full proofs on the exam, with several smaller tasks. There is ONE take-home problem.
2. Given the 50-minute time constraint, you'll find that some correct proofs require only a few sentences.
3. Some statements may be similar to examples you've already seen, but others will not be (else I'm only testing your ability to imitate, rather than actually create your own proof).
4. By their very nature, proof exams require creativity.
   (a) When required to be creative under a time constraint, it's natural to feel lost, rushed, or even blind-sided. Try not to panic, but keep working to show me as much of your skills as possible.
   (b) I like to give partial credit, so include as many *meaningful* ideas as possible, even if you're stuck.
   (c) Putting the "exit move" and other final sentences on the end typically earns some credit, regardless of whether you have a hole in the middle of your proof.
   (d) Again, practice *A LOT* (there are plenty of problems left in the text!), so that you feel comfortable choosing a proof type AND making some progress on any statement you're asked to prove.
5. If you feel you can't finish in time, **DON'T PANIC**. I often take completion time into account when determining whether an exam deserves a curve or a bonus.
6. **DO YOUR BEST** in the exam period, and let me worry about the time factor.
7. Always remember: I like to give partial credit.