Math 235 - Dr. Miller - *Preliminary* Topics List for Exam #3, Spring 2024 - **Apr. 19 OR 22, 2024**

Prepare by studying the topics listed below, in conjunction with your notes, text/readings, any handouts, Short Practice, and graded and ungraded HW problems. You will be more successful if you strive to master the concepts in general, rather than simply memorizing specific examples that we have already done. Studying together is a plus.

Skills from Exam #1-2 and Discrete Math are still needed throughout the course. These include:

1. Identify hypothesis, conclusion in a conditional statement that's written using: if-then, trailing if, only if.
2. Write contrapositives. Recognize "if" versus "only if" directions in biconditional statements.
3. Negate: conditional, and, or, for all/each/every/any, there exists/is, at least one, greater/less/equal, etc.
4. Translate between conditional and universal statements as you find useful.
5. Be familiar with the notations $\mathbf{Z}$, $\mathbf{Z}^+$, $\mathbf{Z}^-$, similar variations for $\mathbf{Q}$ and $\mathbf{R}$.
6. Use FORMAL definitions/statements: rational, even, odd, Division Algorithm, divides, FTA, intervals.
7. Know synonyms for divides (factor, multiple, divisor, divisible by) and how remainders work.

Non-Proof Tasks: Remember that formal statements/definitions have hypotheses. Don't forget to write them!

1. Formally state definitions of: domain, range, reflexive, symmetric, transitive, function, onto, one-to-one.
2. Find elements related/not related to each other for a given relation, including:
   (a) Domain or codomain are sets of numbers, power sets, Cartesian products.
3. Create or analyze sets of ordered pairs or digraphs to have/lack different combinations of the reflexive, symmetric, transitive properties. Prepare to justify beyond just stating the formal definition.
4. Complete sets of ordered pairs or digraphs to have/lack combinations of such properties.
5. Create or analyze sets of ordered pairs or arrow diagrams to be/not be functions, one-to-one, and/or onto. Prepare to justify beyond just stating the formal definition.
6. Identify a given set-up as a function/not, one-to-one/not, onto/not as in multiple choice HW.
7. Functions and relations may be defined using any of these earlier or familiar concepts:
   (a) Parity, rationality, remainders, divisibility, other concepts we've used in this class
   (b) Algebra/pre-calc concepts such as equation/inequality solving, equations of lines/circles, functions, factoring, etc.
   (c) Math terms and concepts from childhood courses or settings: fractions, decimals, geometry, etc.
8. Remember these closure axioms:
   (a) $\mathbf{Z}$ is closed under $+$, $-$, $\times$, and also non-negative exponentiation.
   (b) $\mathbf{R}^+$ is closed under $+$, $\times$, $\div$.
   (c) $\mathbf{R}$ is closed under $+$, $-$, $\times$, non-zero $\div$.
   (d) You must use the formal definition of $\mathbf{Q}$ instead of assuming it is closed in any way.
9. State the Principle of Mathematical Induction (PMI).

Partial Proof Tasks: "Launches" There may be fewer tasks like this than on earlier exams.

1. Show just the START of a proof by giving three pieces of info: assumptions, NTS, and ONE additional sentence.
   (a) I will specify whether you're to go one sentence beyond a first, second, or even third NTS.
2. Remembering all previous styles may be helpful, but I will NOT be forcing specific styles on this exam:
   (a) Conditional statements: Direct (including cases), by contrapositive, or by "or conclusion" style.
   (b) Any statement: by contradiction (so remember how to negate EVERYTHING)
   (c) "For all" proofs, constructive "there exists" proofs
      i. For the constructive proof, you don't need to know the exact candidate; you can just write "Consider t" LITERALLY as your assumption and then move to NTS.
   (d) **NEW:** A disproof of any statement
   (e) A subsetness, set equality, or not-a-subset proof
   (f) The SEPARATE directions of a biconditional statement directly, by ctp, or by contradiction

3. Statements here may use unfamiliar settings, but don't panic - these are simply launches, not full proofs.
4. Statements will primarily focus on domain, range, RST, onto, one-to-one. PMI will NOT be included here.

Full Proof Tasks: Write COMPLETE proofs about relations and functions; write PMI proofs.

1. Emphasis will be on proofs, disproofs of: reflexive, symmetric, transitive properties, of one-to-one, onto.
2. There may also be set proofs about domain, range, composition.
3. Your skills in earlier styles are still needed, but remember that I am being more flexible and forgiving about requiring multiple NTS statements in relation and function proofs.
   (a) You should still be able to navigate cases, including WLOG, "or conclusion" style, contrapositive, contradiction, "for all," constructive "there exists," and disproof.
4. Styles may end up mixed now, as in our RST, one-to-one, onto proofs and disproofs.
   (a) We've also seen instances where a proof by cases has ONE case that leads to a contradiction.
5. Remember that your NTS line or use of definitions can help you continue when stuck.
6. In a constructive "there exists" proof, your candidate might be a concrete value, or it might be a formula.
7. Beware algebra mistakes. Remember that I like to give partial credit.
8. **PMI proofs will appear only on the take-home portion of the exam (1-2 proofs).**
9. Remember these proof components:
   (a) You MUST write your explicit assumptions at the outset, including universal hypotheses.
   (b) You MUST write a formal NTS claim; a "meaning" add-on is helpful but not required.
   (c) You MUST be clear that you have proved what was asked, via the "exit move" sentence.
   (d) Write in SENTENCES. Proofs are never just a string of equalities or algebraic expressions. Such things must appear in context with words telling what they represent/what you're doing with them.
10. I give PARTIAL CREDIT if you have the framework correct for your proof, even if you can't do anything past the NTS. So put on the "Thus" and "exit move" ending sentences even if you're stuck on the middle.

General Advice:

1. There will be 3-4 full proofs on the exam, with several smaller tasks.
2. Given the 50-minute time constraint, you'll find that some correct proofs require only a few sentences.
3. Some statements may be similar to examples you've already seen, but others will not be (else I'm only testing your ability to imitate, rather than actually create your own proof).
4. By their very nature, proof exams require creativity.
   (a) When required to be creative under a time constraint, it's natural to feel lost, rushed, or even blind-sided. Try not to panic, but keep working to show me as much of your skills as possible.
   (b) I like to give partial credit, so include as many *meaningful* ideas as possible, even if you're stuck.
   (c) Putting the "exit move" and other final sentences on the end typically earns some credit, regardless of whether you have a hole in the middle of your proof.
   (d) Again, practice *A LOT* (there are plenty of problems left in the text!), so that you feel comfortable choosing a proof type AND making some progress on any statement you're asked to prove.
5. If you feel you can't finish in time, **DON'T PANIC**. I often take completion time into account when determining whether an exam deserves a curve or a bonus.
6. **DO YOUR BEST** in the exam period, and let me worry about the time factor.
7. Always remember: I like to give partial credit.