<u>Math 235 - Dr. Miller - Topics List for Final Exam, Spring 2024 - **Thursday, May 24, 1:00-3:00 pm**</u>

Prepare by studying these topics, in conjunction with all course materials and reading. Studying together is a plus.

<u>Non-Proof Tasks:</u> Be familiar with set names $\mathbf{Z}$, $\mathbf{Z}^+$, $\mathbf{Z}^-$, similar variations for $\mathbf{Q}$, $\mathbf{R}$; also be clear about intervals.

1. Identify hypothesis, conclusion in conditionals using: if-then, trailing if, only if. Write contrapositives.
2. Recognize "if" vs "only if" direction in biconditional statements, and choice of $\Leftarrow$ vs $\Rightarrow$.
3. Negate: conditional, and/or, for all, there exists, greater/less/equal/3-way inequalities.
4. Identify remainders when we divide by a specific number; use them to create cases in proofs.
5. State FORMAL defns: rational, even, odd, divides, $A \subseteq B$, $A \nsubseteq B$, $A \cup B$, $A \cap B$, $A \setminus B$, $A \times B$, A^c, $n\mathbf{Z}$, $\mathcal{P}(S)$, domain, range, reflexive, symmetric, transitive, function, onto, one-to-one.
   (a) Formal definitions use variables, VERBAL quantifiers,buffer words, and universal hypotheses.
6. Know the different synonyms within divisibility: divides, factor, multiple, divisor, divisible by.
7. Formally state these theorems: Division Algorithm, FTA, PMI, PSI, WOP.
8. Write the logical equivalences governing proof by: cases, "or conclusion"-style, proof by ctp, 2-part.
9. Create or analyze sets of ordered pairs or digraphs to have/lack different combinations of the reflexive, symmetric, transitive properties. Prepare to justify beyond just stating the formal definition.
10. Create or analyze sets of ordered pairs or arrow diagrams to be/not be functions, one-to-one, and/or onto. Prepare to justify beyond just stating the formal definition.

<u>Partial Proof Tasks: "Launches"</u> There will likely be several problems like this.

1. Only START a proof by giving three pieces of info: assumptions, NTS, and ONE additional sentence.
   (a) I will specify whether you're to go one sentence beyond a first, second, or even third NTS.
2. I may ask you to do so using MY choice or allow YOURS among these styles/ types of statements:
   (a) Conditional statements: Direct (including cases), by contrapositive, or by "or conclusion" style.
   (b) Any statement: by contradiction (so remember how to negate EVERYTHING)
   (c) "For all" proofs, constructive "there exists" proofs
      i. For $\exists$ proof, you can write "Consider t" LITERALLY as your assumption and then move on.
   (d) A subsetness, set equality, not-a-subset proof, or set identity (two-part or "chain-style")
   (e) The SEPARATE directions of a biconditional statement directly, by ctp, or by contradiction
      i. Remember: I refer to these as "if direction" and "only if direction," rather than using notation.
   (f) A uniqueness proof; also a disproof of any statement
   (g) A proof of reflexive/symmetric/transitive properties, of one-to-one/onto/function
3. Given a "proof" launch, name the kind of proof attempted or incorrect. I'll give options to choose from.
4. Statements to launch may be about unfamiliar concepts, but don't panic - these are NOT full proofs!

<u>Proof Tasks:</u> Some problems require you to use MY choice of proof type; others may be left to YOUR choice.

1. Remember that your NTS line or use of definitions can help you continue when stuck.
2. Styles may end up mixed now, as in our RST, one-to-one, onto proofs and disproofs.
   (a) We've also seen instances where a proof by cases has ONE case that leads to a contradiction.
   (b) Remember that in such cases I'll be generous about multiple NTS.
3. Be able to navigate all these proof styles/types of statements:
   (a) Direct, cases (including "wlog"), "or conclusion," contrapositive, contradiction, 2-part
   (b) Quantifier proofs: "for all," constructive "there exists," both quantifiers mixed, DISPROOF
      i. In a constructive "there exists" proof, your candidate might be a concrete value vs formula.
   (c) Subset/not/= set proofs, set identities (two-part or chain-style) using $\cup$, $\cap$, $\setminus$, $\times$, complement.
   (d) General set proofs using $\subseteq$, $\nsubseteq$, $=$, $\cup$, $\cap$, $\setminus$, $\times$, complement, power set.
   (e) Proofs, disproofs: reflexive, symmetric, transitive, function, one-to-one, onto.
4. Proofs can include the following concepts (but not limited to these):
   (a) Parity, rationality, remainders, divisibility, other concepts we've used in this class
   (b) Algebra/pre-calc such as equation/inequality solving, equations of lines or circles, functions, etc.
   (c) Math terms, concepts from other courses or settings: fractions, decimals, geometry, e^x and $\ln x$, etc.
   (d) Relations/functions with domains that are sets of numbers, power sets, Cartesian products, etc.
5. Beware algebra mistakes. Remember that I like to give partial credit.
6. There MAY be one short PMI proof about divisibility or inequality. We'll discuss in class.