Math 118 - Dr. Miller - Homework #4: Perimeters and Polyominos

1. If you haven't already done so, complete the in-class Activity about polyominos. Remember that a polyomino diagram that just looks like you flipped another one over, or spun another one around, does NOT count as a differently shaped object.

2. There are 12 distinctly shaped pentominoes. Draw them all.

3. Draw a hexomino that has minimum perimeter; tell how you know this without drawing all possible hexomino shapes.

4. Draw three distinctly shaped hexominoes that all have the maximum perimeter possible. Again, tell how you know.

5. Draw a heptomino (a polyomino using 7 squares) that has the minimum possible perimeter, then one that has the maximum possible. Can you draw one whose perimeter falls in between the minimum and the maximum? If so, do it.

6. None of the quadrominoes in the Activity have perimeters that are odd numbers. Do any pentominoes have odd perimeters?

7. Do you think that there are any hexominoes that have odd perimeters? Explain, using mathematically correct ideas.

8. Create a table listing the number of squares in each type of polyomino and the maximum perimeter obtainable from polyominos of that type. Is it possible to predict the maximum perimeter possible when all you know is the number of squares in the polyomino? If so, describe how to do so.

9. Create a table listing the number of squares in each type of polyomino and the minimum perimeter obtainable from polyominos of that type. Is it possible to predict the minimum perimeter possible when all you know is the number of squares in the polyomino? If so, describe how to do so.

Math 118 - Dr. Miller - Solutions to HW #4: Perimeters and Polyominos

1. Answers to the Activity questions:
   (a) The quadrominoes are the shapes familiar from the Tetris video game. They all have perimeter equal to 10 except for the perfect square, whose perimeter is 8.
   (b) Various answers are possible.
   (c) The pentomino shown is the only one that is compacted to resemble a square as much as possible, so it has the smallest possible perimeter, and the other more stretched-out pentominoes cannot also have that small a perimeter.
2. Check the online diagrams.
3. A 2-by-3 rectangle is the most "square-like" of all hexominoes, so it has the smallest possible perimeter.
4. Various answers, with the long, skinny 1-by-6 rectangle among them. It is most "stretched out."
5. One diagram for the minimum possible perimeter is a 2-by-3 rectangle with one extra square attached. Its perimeter is the minimum possible: 12. An easy one having the maximum possible perimeter is a 1-by-7 rectangle. It is most stretched out, so its perimeter is the maximum: 16. You can achieve a perimeter of 14 by taking a 2-by-2 square and attaching a 1-by-3 "tail" sticking out from it.
6. No. The pentominoes have perimeters of 10 or 12 only.
7. No. No polyomino will ever have an odd perimeter. Thinking in child-like terms, the perimeter is the distance you travel when walking around a shape. Imagine fixing a starting point and then doing this. For each horizontal step that you EVER take moving farther away from your start, you'll have to cover that distance (NOT that exact path, but certainly that distance) going back, no matter how you "snake" backwards and forwards in walking around the shape. So every horizontal step away is paired with a horizontal step returning home, giving you an even total horizontally. The same is true vertically, and so the total of horizontal and vertical distance traveled is also even every time.
8.

Name	*No. of Squares*	*Maximum Perimeter*
Unomino	1	4
Domino	2	6
Tromino	3	8
Quadromino	4	10
Pentomino	5	12

The pattern shows the maximum perimeters increasing regularly: they are even whole numbers, in order, but we want to predict more accurately than just counting by 2s: if we double the number of squares and add 2, we always get the maximum possible perimeter. That's because the maximum comes from long, skinny rectangles that are 1 unit tall and as long as our number of squares, say n. So the total perimeter is always $1 + n + 1 + n = 2n + 2$.

9.

Name	*No. of Squares*	*Minimum Perimeter*
Unomino	1	4
Domino	2	6
Tromino	3	8
Quadromino	4	8
Pentomino	5	10
Hexomino	6	10
Heptomino	7	12
Octomino	8	12

There's not much of a pattern in the beginning: we count by even whole numbers but then "stall" at 8, then again at 10. It begins to look, though, as if , after, just one 4 and one 6, the pattern will list each even number twice: 8,8, 10,10, 12, 12, ... But that's not right. The very next polyomino in the list, with 9 squares in it, has a minimum perimeter of 12, not 14. So the bottom line is that we can't easily predict what happens next, but it would make a great project to explore!