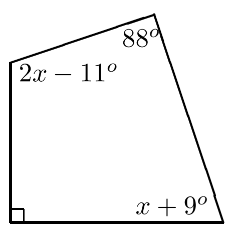
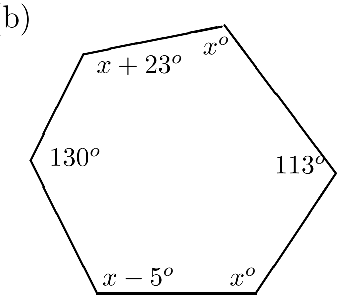
1. (a) How many diagonals does a dodecagon have per vertex?
   (b) How many does it have altogether?
2. (a) Is it possible for a polygon to have 66 diagonals per vertex? Explain.
   (b) Is it possible for a polygon to have 66 diagonals altogether? Explain.
3. How many sides does a polygon having 405 diagonals have?
4. (a) What is the sum of the interior angle measures of a 40-sided convex polygon?
   (b) Repeat the above for a 100-sided convex polygon.
5. The total interior angle measure of a certain convex n-gon is 6480^o. How many sides does it have?
6. The interior angle total for a certain convex n-gon is 7,560. What is the value of n?
7. What is the measure of a single interior angle of a regular dodecagon?
8. What is the measure of a single interior angle of a regular 50-gon?
9. (a) Each interior angle of a certain regular polygon measures 160^o. How many sides does it have?
   (b) Repeat the above question for 174^o.
10. Find the missing angle measures; round to the nearest hundredth.

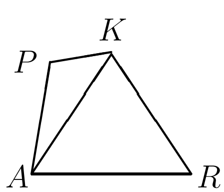
    (a)



    (b)



11. Find the measurement of each vertex angle of a regular pentagram, including the reflex angles. (A pentagram is the five-pointed star we all learn to draw as children.)
12. Find the measurement of $\angle PAR$, given that $m(\angle P) = 110^o$, $m(\angle PKR) = 108^o$, and $\triangle AKR$ is regular.



1. (a) It has $10 - 3 = 7$ diagonals per vertex.
   (b) A dodecagon has $(12)(9) \div 2$, or 54, diagonals.

2. (a) Yes: a polygon with 69 sides would have 66 diagonals per vertex, because that vertex can form diagonals with all but three of the 69 total vertices (can't make diagonals with itself or the two adjacent vertices)
   (b) A 13-gon has $(13)(10) \div 2$, or 65 of them; a 14-gon has $(14)(11) \div 2$, or 77. We've skipped over the possibility of 66, so it can't occur.

3. 405 diagonals total means twice as many – 810 – when we multiplied $n \times (n - 3)$. Guess and check:

$60 \cdot 57 = 3420$ way too high!

$20 \cdot 17 = 340$ too low

$30 \cdot 27 = 810$ that's it! Answer: 30 sides.

4. (a) The sum uses the formula $(n - 2) \cdot 180^o$, so this total is $38 \cdot 180$, or $6,840^o$.
   (b) This total is $98 \cdot 180$, or $17,640^o$.

5. Note that 6480 must equal $n - 2$ times 180. Dividing 6480 by 180 gives 36, but that's $n - 2$, so n is 38.

6. Here, 7560 must equal $n - 2$ times 180. Dividing 7560 by 180 gives 42, but that's $n - 2$, so n is 44.

7. In a *regular* polygon, each angle gets an equal share of the total. The total here is $10 \cdot 180$, or 1800^o; shared equally among the 12 angles, that's 150^o apiece.

8. The total is $48 \cdot 180$, or $8,640^o$. Each of the 50 angles gets an equal share, so that's 172.8^o apiece.

9. (a) *Guess and check solution:*
   From the answers to Problems #7 and #8, n is somewhere between 12 and 50.
   Try $n = 30$. Then the total is $28 \cdot 180 = 5040^o$, shared as 168^o each. *Too big.*
   Try $n = 20$. The total is $18 \cdot 180 = 3240^o$, shared as 162^o apiece. *Still too big.*
   Try $n = 18$. The total's $16 \cdot 180 = 2880^o$, or 160^o each. *That's it!*
   Algebraic solution:

$$\begin{aligned} 160 &= \frac{(n-2)\cdot 180}{n} \\ 160n &= (n-2)\cdot 180 \\ 160n &= 180n - 360 \\ 360 &= 20n \qquad \textit{so } n = 18 \end{aligned}$$

   (b) *Guess and check solution:*
   This time, we know there are more than $n = 50$ sides from looking at part (b).
   Try $n = 80$. The total is $78 \cdot 180 = 14040^o$, or 175.5^o each. *That's too high.*
   Try $n = 70$. Now the total's $68 \cdot 180 = 12240^o$, for 174.9^o each. *Still too high.*
   Try $n = 60$. The total's $58 \cdot 180 = 10440$, for 174^o on the nose! *Algebraic solution:*

$$\begin{aligned} 174 &= \frac{(n-2)\cdot 180}{n} \\ 174n &= (n-2)\cdot 180 \\ 174n &= 180n - 360 \\ 360 &= 6n \qquad \textit{so } n = 60 \end{aligned}$$

10. (a) $x = 61.3$, so the missing angles are 111.6^o and 70.3^o.
    (b) $x = 114.75$, so the missing angles are 137.75^o, 109.75^o, and two worth 114.75^o.

11. The acute vertex angles measure 36^o each; the reflex ones measure 252^o each.

12. $m(\angle PAR) = 82^o$.