Math 118 - Dr. Miller - Homework \#24: Advanced Triangle Congruences
Consider this diagram:


Apply the given information to find a pair of congruent triangles. Use correct notation to tell which two triangles they are and specify the acronym you use, then thoroughly explain how you arrived at your conclusion.

1. Assume nothing except that $A B D E$ is a rhombus.
2. Assume nothing except that $\overleftrightarrow{F B} \| \overleftrightarrow{E C}$ and $\overline{B X} \cong \overline{E Y}$.
3. Assume nothing except that $\overline{A E}$ and $\overline{F B}$ bisect each other.
4. Assume nothing except that $A C D F$ is a rectangle and $\overline{E F} \cong \overline{A B}$.
5. Assume nothing except that $\overleftrightarrow{F D} \| \overleftrightarrow{A C}$ and $X$ is the midpoint of $\overline{A E}$.
6. Assume nothing except that $\overleftrightarrow{B E}$ is the perpendicular bisector of $\overline{A C}$.
7. Assume nothing except that $\overline{A B} \cong \overline{A F}$ and $\overline{B F}$ and $\overline{A E}$ bisect each other.
8. Assume nothing except that $\angle E X B \cong \angle B Y E$ and that $\overline{B E}$ bisects $\angle F B D$.

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1. $\overline{A B} \cong \overline{B D}$ because they're sides of the rhombus.
$\overline{A E} \cong \overline{E D}$ because they're also sides of the rhombus.
Side $\overline{E B}$ is shared.
So $\triangle A B E \cong \triangle D E B$ by SSS.
2. $\overline{B X} \cong \overline{E Y}$ because we're told so.
$\angle F B E \cong \angle C E B$ because they're alternate interior angles between $\overleftrightarrow{F B}$ and $\overleftrightarrow{E C}$.
Side $\overline{B E}$ is shared.
So $\triangle X B E \cong \triangle Y E B$ by SAS.
3. $\overline{A X} \cong \overline{X E}$ because $\overline{A E}$ is bisected.
$\overline{F X} \cong \overline{X B}$ because $\overline{F B}$ is also bisected.
$\angle F X E \cong \angle A X B$ because they're vertical angles. (So are $\angle F X A a n d \angle B X E$.)
So $\triangle F X E \cong \triangle B X A$ by SAS. (So are $\triangle F X A$ and $\triangle B X E$.)
4. $\overline{E F} \cong \overline{A B}$ because we're told so. $\angle F A B \cong \angle A F E$ because they're both right angles (in the rectangle).
Side $\overline{F A}$ is shared.
So $\triangle A F E \cong \triangle F A B$ by SAS.
5. $\overline{A X} \cong \overline{E X}$ because $\overline{A E}$ is bisected.
$\angle F X E \cong \angle A X B$ because they're vertical angles.
$\angle F E A \cong \angle B A E$ because they're alternate interior between $\overleftrightarrow{F B}$ and $\overleftrightarrow{E C}$.
So $\triangle F E X \cong \triangle B A X$ by ASA.
6. $\overline{A B} \cong \overline{C B}$ because $\overline{A C}$ is bisected.
$\angle A B E \cong \angle C B E$ because they're both right angles (due to perpendicularity).
Side $\overline{B E}$ is shared.
So $\triangle A B E \cong \triangle C B E$ by SAS.
7. $\overline{A B} \cong \overline{A F}$ because we're told so.
$\overline{F X} \cong \overline{B X}$ because $\overline{B F}$ is bisected.
Side $\overline{A X}$ is shared.
So $\triangle A X F \cong \triangle A X B$ by SSS.
8. $\angle E X B \cong \angle B Y E$ because we're told so.
$\angle F B E \cong \angle D B E$ because $\angle F B D$ is bisected.
$\angle A E B \cong \angle Y E B$ because they're the third angles of the triangles.
Side $\overline{B E}$ is shared.
$\triangle B E X \cong \triangle B E Y$ by ASA.
