Math 118 - Dr. Miller - Activity #20: Arrangements of Lines

*Work together, and turn the attached* **Quiz #15** *in to my mailbox when you're done. This material will appear on the Final Exam, but NOT on Exam #3.*

Recall that "the plane" refers to a flat surface that extends infinitely in all directions, and that lines in a plane also extend infinitely. We can talk about how many regions the plane is divided into by the lines by imagining that we use "infinite scissors" to cut along those lines, and then count the number of separated pieces made. The lines themselves don't count as regions or pieces; only the separate spaces between the lines do.

1. *Concurrent Lines:*

   (a) Two intersecting lines divide the plane into four regions. Draw a sketch and number these regions.

   (b) Into how many regions do 3 *concurrent* lines divide the plane? Again, draw a sketch and number the regions.

   (c) Repeat the above for 4 concurrent lines.

   (d) Make a conjecture for the number of regions 5 concurrent lines would create.

   (e) Make a conjecture for how you could predict the number of regions any particular number (say, $n$, if you like) of concurrent lines will create.

2. *Parallel Lines:*

   (a) Into how many regions do 2 parallel lines divide the plane? Sketch and number as in Problem #1.

   (b) Into how many regions do 3 parallel lines divide the plane? Sketch and number.

   (c) Predict how many regions 4 parallel lines divide the plane into, then 5 parallel lines.

   (d) Make a conjecture for how to predict the number of regions any number (say, $n$) of parallel lines would create.

3. *"Generic" Lines:*

   When we consider the behavior of just 2 lines in a plane, we know that they will either intersect or be parallel; there are no other options. However, when we consider a higher number such as 3, the lines may all 3 be parallel, they may all 3 be concurrent, or they may intersect in a more "off-set" fashion and form the outline of a triangle. Let's say that when we have a collection of lines, none of which are parallel nor concurrent, we'll call them "*generic*."

   (a) Into how many regions do 3 generic lines divide the plane? Sketch and number.

(b) Into how many regions do 4 generic lines divide the plane? Sketch and number.

(c) Repeat for 5 generic lines. You'll need a very clear sketch.

(d) Verify that the formula $1 + \frac{n(n+1)}{2}$ predicts the number of regions formed by $n = 3$, $n = 4$, and $n = 5$ generic lines. (Hint: your picture for 5 generic lines should have had a 5-pointed star in it.)

4. "Tweener" Numbers of Regions

We've seen that 2 lines divide the plane into 3 regions if they're parallel and 4 regions if they're generic (make sure you agree that for the case of just 2 lines, generic and intersecting mean the same thing); there's no other number of regions between 3 and 4. We've also seen that 3 lines divide the plane into 4 regions if they're parallel but 7 if they're generic; what about numbers of regions between 4 and 7...?

(a) Is it possible to sketch 3 lines that divide the plane into 5 regions? Into 6?

(b) Four lines divide the plane into 5 regions at one extreme and 11 at the other. Is it possible to sketch 4 lines dividing the plane into 6 regions? 7? 10?

(c) Five lines divide the plane into 6 regions if they're parallel; can you sketch them so that they create 7 regions? Can you position six lines to make 8 regions?

(d) Make a prediction about positioning $n$ lines to make just one more region than they would if parallel.

(e) Five lines divide the plane into 16 regions if they're generic; sketch them so that they create just 15.

(f) Make a prediction about positioning $n$ lines to make just one less region than they would if generic. (I'll answer this one for you.)

*Answer: Take the generic lines, and turn ONE of them so it will be parallel to another. We only lose one region by doing this: the "infinite triangle" that used to be between the two lines when they crossed, but now don't.*

Some Review Questions - **Quiz #15 - LAST ONE**

1. Cutting a plane along three lines breaks it into how many pieces? Give all possible answers; for each, draw a sketch and number your regions.

2. (a) Cutting a plane along four parallel lines breaks it into _____ pieces.

   (b) Cutting a plane along $n$ parallel lines breaks it into _____ pieces.

3. (a) Cutting a plane along four concurrent lines breaks it into _____ pieces.

   (b) Cutting a plane along $n$ concurrent lines breaks it into _____ pieces.

4. How many concurrent lines would you need...

   (a) ...if you wanted to cut the plane into 18 regions?

   (b) What if you wanted to make 108 regions?

   (c) What if you wanted to make 107 regions?

5. How many parallel lines would you need...

   (a) ...if you wanted to cut the plane into 18 regions?

   (b) What if you wanted to make 108 regions?

   (c) What if you wanted to make 107 regions?

6. How should you arrange 7 lines to make...(answer with a word or verbal description)

   (a) 8 regions?

   (b) 9 regions?

   (c) 29 regions? (answer with a word or verbal description, not a picture)

   (d) 28 regions? (answer with a word or verbal description, not a picture)

7. Can you cut the plane into 57 regions using generic lines? Justify your claim.

8. You need to cut a round cake into 30 slices at a party. What types of lines should you cut along, and how many cuts must you make?

9. Lowe's will cut boards for you at 25 cents per cut. You need to make 6 pieces out of a board you bought. What types of lines should they cut along, and how many cuts will you pay for?