

1. How many 8-letter codes start or end with either AB or CD?
2. How many 8-letter codes use only even digits or end in 0?
3. How many 3 letter codes have exactly 1 A or exactly 2 Cs?
4. How many 3 letter codes have only vowels or only letters later than M in the alphabet?

Solutions:

1. Start - $2 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 1440$
end - $2 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 1440$
do both - $2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 48$
Answer = $1440 + 1440 - 48 = 2832$
2. Only even - $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^8$
end in 0 - $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 1 = 10^7$
do both - $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 1 = 5^7$
Answer = $5^8 + 10^7 - 5^7 = 390625 + 10,000,000 - 78125 = 10,312,500$
3. Exactly 1 A - $3 \cdot 1 \cdot 25 \cdot 25 = 1875$
exactly 2 Cs - $3 \cdot 1 \cdot 1 \cdot 25 = 75$
do both - 3 listed as ACC, CAC, or CCA
Answer = $1875 + 75 - 3 = 1947$
4. Only vowels - $5 \cdot 5 \cdot 5 = 125$
only letters later than M - $13 \cdot 13 \cdot 13 = 2197$
do both - $2 \cdot 2 \cdot 2 = 8$
Answer = $125 + 2197 - 8 = 2314$