1. Symbolically negate the following statements\((P, Q, \text{ and } R \text{ are statement variables.})\)

(a) \(\forall x, [P \lor (Q \implies R)]\)

\(\exists x, [\sim P \land (Q \land \sim R)]\)

(b) \((\forall x, P) \lor (Q \implies R)\)

\((\exists x, \sim P) \land (Q \land \sim R)\)

(c) \(\exists x, [P \implies (Q \land R)]\)

\(\forall x, [P \land (\sim Q \lor \sim R)]\)

(d) \(\exists x, (\exists y, P) \implies (Q \land R)\)

\(\forall x, (\exists y, P) \land (\sim Q \lor \sim R)\)

\textit{Tricky!} \(\forall x, (\exists y, P) \land (\sim Q \lor \sim R)\)

2. Symbolically negate the following statements. Be careful with domains.

(a) \(\forall x > 0, x^2 > 0 \land x + 1 > 0\).

\(\exists x > 0, x^2 \leq 0 \lor x + 1 \leq 0\)

(b) \(\forall x \leq 10, x \leq 0 \implies x^2 > x\).

\(\exists x \leq 10, x \leq 0 \land x^2 \leq x\)

(c) \(\exists x > 0, x^2 < x \land x < 0.25\)

\(\forall x > 0, x^2 \geq x \lor x \geq 0.25\)

(d) \(\exists x \leq 8, \forall y > 0, xy < y\)

\(\forall x \leq 8, \exists y > 0, xy \geq y\)

3. Verbally negate these mathematical statements. Use correct mathematical grammar.

(a) For all positive real numbers \(r\), \(3r + 1 > r\) and \(r - 1 \leq r\).

\textit{There is a positive real number \(r\) for which \(3r + 1 \leq r\) or \(r - 1 > r\).}

(b) There exists a positive integer \(x\) for which \(x^2 > 3\) or \(x - 8 < 0\).

\textit{For all positive integers \(x\), \(x^2 \leq 3\) and \(x - 8 \geq 0\).}

(c) For all real numbers \(z\) and \(w\), \(z + w > 10\) implies that \(z > 5\) or \(w > 5\).

\textit{There are real numbers \(z\) and \(w\) for which \(z + w > 10\) but \(z \leq 5\) and \(w \leq 5\).}

(d) There exist positive integers \(x\) and \(y\) for which \(xy - 10\) is negative.

\textit{For all positive integers \(x\) and \(y\), \(xy - 10\) is positive or 0.}

(e) There exists a real number \(M\) for which \(Mx > x\) for all positive real numbers \(x\).

\textit{For all real numbers \(M\), \(Mx \leq x\) for some positive real number \(x\).}
(f) For every real number \( r \neq 0 \), there exists an integer \( n \) with \( n > 1/r \).

There is a real number \( r \neq 0 \) (keep same domain!), for which, for all integers \( n \),
\( n \leq 1/r \).

(g) There exist rational numbers \( p \) and \( q \) for which \( \sqrt{pq} \) is rational but \( \sqrt{p^2q} \) is irrational.

For all rational numbers \( p \) and \( q \), \( \sqrt{pq} \) is irrational or \( \sqrt{p^2q} \) is rational.

(h) There exists an integer \( x > 10^{100} \) that is prime and \( x = 2^n+1 \) for some integer \( n \).

For all integers \( x > 10^{100} \), \( x \) is not prime (is composite in such a case) or \( x \neq 2^n+1 \)
for any integer \( n \). (The “any” is a smoother grammatical synonym for “all” in this setting.)

(i) For every \( \epsilon > 0 \), there exists a \( \delta > 0 \) for which \( |x-c| < \delta \) implies that \( |f(x)-L| < \epsilon \).

There is \( \epsilon > 0 \) for which for all \( \delta > 0 \), \( |x-c| < \delta \) but \( |f(x)-L| \geq \epsilon \).

Surprisingly, it is not always the case that negating \( > \) simply gives \( \leq \). In
the problems above, however, I consider that to be a fair assumption on
your part. Sometimes in math, though, we encounter situations where
a value doesn’t always exist and so can’t be described as greater, less,
or equal to some actual number. The function \( \tan(x) \) is an example; you
know that for some values of \( x \), \( \tan(x) \) has a 0 denominator and so isn’t
defined/doesn’t create a numerical value. In that case, simplifying the
negation of \( \tan(x) > 0 \), that is, saying that \( \tan(x) \not< 0 \), would actually have
to account for that weird case of non-numeric behavior too. Thus, the
negation of \( \tan(x) > 0 \), barring any special circumstances, would actually
be “\( \tan(x) \leq 0 \) or \( \tan(x) \) is undefined” as we think about the full context
of the statement as well as its logical form.

4. Informally negate the following verbal statements:

(a) Everybody loves Raymond.

\textit{Negation:} Somebody doesn’t love Raymond. (“hate” is an opposite, not a negation.)

(b) Nobody likes Chris.

\textit{Negation:} Somebody likes Chris.

(c) Tomorrow never dies.

\textit{Negation:} Tomorrow sometimes dies.

(d) Some days, you don’t want to get out of bed.

\textit{Negation:} Every day, you do want to get out of bed.

(e) Everybody wants to rule the world.

\textit{Negation:} Somebody doesn’t want to rule the world.

(f) There’s a new day coming.

\textit{Negation:} (We can do better than “There isn’t.”) All days coming aren’t new.
All new days aren’t coming.

(g) All dogs go to heaven.
   Negation: Some dogs don’t go to heaven. (“Hell” is an opposite.)

(h) I have always had to rely on the kindness of strangers.
   Negation: I have sometimes not had to rely on the kindness of strangers.

(i) I’ll never be hungry again.
   Negation: I’ll sometime be hungry again. I’ll be hungry again sometime.

(j) Some like it hot.
   Negation: All don’t like it hot. (“Cold” is an opposite.)

(k) I always feel like somebody’s watching me.
   Negation: I sometimes feel like no one’s watching me. (Accepted: I sometimes don’t
   feel like somebody’s watching me.)

(l) Every breath you take, I’ll be watching you.
   Negation: Some breaths you take, I won’t be watching you.

(m) Some people call me Maurice.
   Negation: Nobody calls me Maurice. Everybody doesn’t call me Maurice.

(n) You win some (and) you lose some.
   Negation: You don’t win any or you don’t lose any.

(o) There’s a lady who’s sure all that glitters is gold.
   Negation: Every lady is not sure all that glitters is gold. (We have to negate the
   quality of our variable, the “lady,” not of the other item, gold, that isn’t even a
   variable. You may have tried “Every lady is sure/not sure that some things that
   glitter aren’t gold,” which is close, but not right.)

(p) (Challenge:) Everybody loves somebody sometime.
   This one is very challenging: “Somebody doesn’t love everybody all the time” is
   ambiguous grammatically. Does it mean there is a person, say Alex, for whom the
   way Alex feels about every single other person in the world all the time is that Alex
   doesn’t love them? Or does it mean (more like daily speech) that if you ask Alex,
   “Do you love everybody?” Alex will always reply “no.” These mean different things.
   For logical negation, we want the former.

5. Work these problems from the textbook:
   p. 82: #1-3, 16-21