Math 131 - Dr. Miller - **SOLUTIONS** to Activity #8: Arbitrary Unions and Intersections - Fall 2024

Find the indicated unions and intersections for the indexed families described below. Show plenty of work for help in studying and solving HW problems.

1. Let the index set $I = \mathbf{Z}^+ \cup \{0\}$ and $A_i = \{i, i-1, i-2, i-3\}$.

   (a) Find $\bigcap_{i=2}^{4} A_i$ and $\bigcup_{i=2}^{4} A_i$. Tell which is which.

   We know that $\bigcap_{i=2}^{4} A_i = A_2 \cap A_3 \cap A_4$ *and* $\bigcup_{i=2}^{4} A_i = A_2 \cup A_3 \cup A_4$. *So we should first find these three individual sets.*

   A_2 *has an i-value of* $i = 2$, *so we substitute* $i = 2$ *into the formula that defines* A_i *in general.* $A_2 = \{2, 1, 0, -1\}$.
   A_3 *has an i-value of* $i = 3$, *so we substitute* $i = 3$ *into the formula:* $A_3 = \{3, 2, 1, 0\}$.
   Finally, A_4 *has an i-value of* $i = 4$, *so we use* $i = 4$ *in the* A_i *formula:* $A_4 = \{4, 3, 2, 1\}$.

   For union, we know to "dump" all the set together and see everything that has accumulated. So $\bigcup_{i=2}^{4} A_i = \{4, 3, 2, 1, 0, -1\}$.
   For intersection, we want to collect ONLY those elements that are in common for ALL THREE sets. So $\bigcap_{i=2}^{4} A_i = \{2, 1\}$.

   (b) Find $\bigcap_{1}^{\infty} A_i$ and $\bigcup_{1}^{\infty} A_i$. Tell which is which.

   Now we have to find the union and intersection for all infinitely many of the sets together.

   $\bigcap_{1}^{\infty} A_i = \emptyset$, because there are no numbers that will be in common/will be shared simultaneously by ALL of the sets at once.

   $\bigcup_{1}^{\infty} A_i = \{-2, -1, 0, 1, 2, 3, \ldots\}$, because when we "dump" all infinitely many sets together, we do collect all integers from the lowest -2, which belongs to A_1, and up.