Math 131-01 - Dr. Miller - Exam #1 - 10/1/01

Answer in the blue book provided. This exam is worth 100 points; point values for each question are indicated in brackets.

1. *[10 pts]* Are the two statements $(p \wedge q) \vee \overline{q}$ and $p \to q$ logically equivalent? Justify your answer with a truth table and a short sentence of explanation.

2. (a) *[4 pts]* Let p = my dog is brown, q = my cat is big, and r = I like pizza. Convert "$(p \vee q) \leftrightarrow \overline{r}$" to a verbal statement.
   (b) *[6 pts]* Convert to completely symbolic form: "For every positive number x, if y is positive, then there is a natural number z for which $xz > y$."

3. *[9 pts - 3 each]* Rewrite the following statements in the form "If ..., then ...."
   (a) You'll only get a cookie if you clean your plate first.
   (b) To get a driver's license, it's necessary to pass the written test.
   (c) $\sqrt{2}$ is an irrational number.

4. Symbolically negate the following, simplifying as much as possible:
   (a) *[4 pts]* $(A > B) \to (A^2 > 3B)$
   (b) *[6pts]* $\exists x, (\forall y, xy > 1)$ and x is even.

5. *[5 pts]* Give a complete, correctly phrased counterexample to the claim: "If a and b are real numbers satisfying $a^2 > b^2$, then $a > b$."

6. Consider: "If x and y are consecutive odd integers, then $xy + 1$ is a perfect square."
   (a) *[5 pts]* If you were going to write a *direct proof* of this result, what would you assume (be explicit), and what would you try to show?
   (b) *[5 pts]* If you were going to write a *proof by contradiction* of this result, what would you assume (be explicit), and what would you try to show?
   (c) *[5 pts]* If you were going to write a *proof by contrapositive* of this result, what would you assume (be explicit), and what would you try to show?

7. *[12 pts]* Prove by using mathematical induction: $3 + 3^2 + 3^3 + \cdots + 3^n = \frac{3(3^n-1)}{2}$.

8. Let the universal set be $U = \{1, 2, 3, 4, 5, 6\}$, and let $A = \{2, 4, 6\}$, $B = \{1, 2, 3\}$, and $C = \{4, 5\}$.
   (a) *[3 pts]* Use the symbol $\subseteq$ correctly in a statement about A.
   (b) *[4 pts]* List the elements of $B \cup (A \cap C)$.
   (c) *[4 pts]* List the elements of $\overline{C} \setminus A$.
   (d) *[3 pts]* List the elements of $\mathcal{P}(C)$, the power set of C.

9. *[8 pts]* Use a Venn diagram with sets P and Q to determine whether $(P \cap Q) \cup \overline{Q} = \overline{P}$.

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10. *Take-Home Question [7 pts]:* (Due Wednesday) Prove the statement from Question #6 directly. You may refer to your notes and text, but **do not** seek help from any other source, especially human. I will severely penalize cheating.