Math 131 - Dr. Miller - Exam #1 - Wednesday, Sept. 28, 2011

Address each problem carefully and thoroughly. This exam is worth 100 points.

1. (a) *[10 pts]* Are the statement forms $q \Rightarrow \sim (p \vee q)$ and $\sim p \vee \sim q$ logically equivalent? Justify your response.

   (b) *[10 pts]* Construct a truth table for the statement form $(p \wedge \sim q) \Rightarrow r$.

2. *[15 pts - 5 each]* For this problem, consider the following statement variables:

$p$: $x$ is prime $\qquad$ $q$: $x$ is odd $\qquad$ $r$: $x > 7$

(a) Rewrite $(r \vee \sim p) \Rightarrow \ \sim q$ verbally.

(b) Rewrite $q \wedge \sim (r \vee p)$ verbally.

(c) Rewrite "$x$ is an even prime only if it's prime or less than or equal to 7" ENTIRELY symbolically, using the given $p$, $q$, and $r$.

3. *[15 pts - 5 each]* Consider the statement "$\sin x > 0$ and $\cos x < 0$ if the angle $x$ is in Quadrant II."

(a) Rewrite the original statement as a universal statement.

(b) Write the converse of the original statement using the phrase "necessary."

(c) Write the inverse of the original statement using the word "sufficient."

4. *[15 pts - 5 each]* Convert each verbal statement to totally symbolic form (no words) and vice versa.

   (a) Some real numbers are rational.

   (b) $x$ is either a rational number but not an integer, or else $x = 0$.

   (c) There is a real number $x$ for which $xy$ is rational for all rational numbers $y$.

5. *[20 pts - 5 each]* Write the negation of each statement below. You may use your choice of verbal or symbolic form.

   (a) There is a real number $x$ for which $xy$ is rational for all rational numbers $y$. (See Problem #4c.)

   (b) Some rectangles are squares.

   (c) If $xy = 0$, then $x = 0$ or $y = 0$.

   (d) All pentagons are polygons.

6. *[15 pts - 5 each]* Identify each statement below as true or false, then justify your claim using an appropriate method.

   (a) Some even integers are prime.

   (b) Any integer $x$ where $48 \leq x \leq 51$ is not prime.

   (c) If 6 is a prime number and 4 is not, then $6 + 4 = 15$.