

Work on the separate sheets provided; attach this question sheet on the front. This exam is worth 100 points.

1. [9 pts - 3 each] Verbally negate each of the statements below:
 - (a) You can have fries or a salad.
 - (b) If you don't like pizza, you're crazy.
 - (c) Every time it rains, my basement floods.

2. (a) [2 pts] Write the converse of "If you don't like pizza, you're crazy."
(b) [3 pts] Write the contrapositive of "A cat is a mammal."

3. [6 pts - 2 each] Convert each statement to the form "(quantifier phrase), (quality)":
 - (a) Squares are polygons.
 - (b) If $x \geq y$, then $x^2 \geq y^2$.
 - (c) $x^2 < x$ for some real number x .

4. [15 pts - 5 each] Symbolically negate each statement below, simplifying your result as fully as possible. Show intermediate steps to receive full credit.
 - (a) $(p \vee q) \rightarrow \neg r$
 - (b) $\forall x \in \mathbf{Q}, \forall y \in \mathbf{Z}, x \leq y$
 - (c) $\exists a \in \mathbf{R}, a > 0$ and $a^2 < a$

5. [8 pts] Is the statement $(p \vee q) \wedge (\neg p \rightarrow q)$ logically equivalent to $p \rightarrow \neg q$? Justify your claim.

6. [6 pts] Reduce the statement $(p \rightarrow q) \vee \neg(p \wedge \neg q)$ to the simplest form you can. Show work.

7. [4 pts] Find the truth value of " $s \wedge (t \vee \neg r) \rightarrow \neg u$ " when s is true, t is false, and $r \wedge u$ is true. Show brief work.

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8. [15 pts - 2 or 3 each] Classify each statement below as true or false:

- (a) $0 \in \{x + y \mid x, y \in \mathbf{Z} \text{ and } x^2 = y^2\}$
- (b) $4 \in \{x + y \mid x, y \in \mathbf{Z} \text{ and } x^2 = y^2\}$
- (c) $1 \in \{x + y \mid x, y \in \mathbf{Z} \text{ and } x^2 = y^2\}$
- (d) $\emptyset \in \mathcal{P}(\{1\})$
- (e) $\emptyset \subseteq \mathcal{P}(\{1\})$
- (f) $\{1\} \subseteq \mathcal{P}(\{1\})$

9. [10 pts - 3 or 4 each] Let the universal set $U = \{5, 6, 7, 8, 9\}$ and let $A = \{x \in U \mid x > 8\}$ while $B = \{5, 7, 9\}$ and $C = \{6, 7\}$. Use correct notation to find the following sets:

- (a) $(A \cup C) \cap \overline{B}$
- (b) $C \setminus B$
- (c) $C \times B$

10. [6 pts] Shade the following set in a three-set Venn diagram. Show intermediate work as needed, but record your final answer in a separate diagram.

$$(A \cap \overline{B}) \cup C$$

11. [8 pts - 2 or 4 each] Let the index set $I = \mathbf{N}$ and let $A_i = \{-1, 1, i\}$.

- (a) Find $\bigcap_{i=3}^5 A_i$. Show work as needed.
- (b) Find $\bigcup_{i=3}^5 A_i$. Show work as needed.
- (c) Find $\bigcap_I A_i$. Show work as needed.

12. [8 pts - 4 each] Let the index set $I = \mathbf{R}^+$, the set of positive real numbers, and let $A_i = [-i, i)$.

- (a) Find $\bigcup_I A_i$. Show work.
- (b) Find $\bigcap_I A_i$. Show work.