Work all problems in the blue book provided. Take this sheet with you when you leave.

1. Let \( s_n = (−1)^n \cdot (2n − 1) \) define a sequence.
   (a) [3 pts] List the first 5 terms of this sequence, beginning with \( s_1 \).
   (b) [3 pts] Is the sequence \( \{s_n\} \) increasing, decreasing, or neither? Explain.
   (c) [2 pts] Evaluate \( \prod_{n=3}^{5} s_{n-1} \).
   (d) [4 pts] Define \( \Omega_k = \sum_{n=1}^{k} s_n \). Compute the values of \( \Omega_3 \) and \( \Omega_4 \).

2. Let a relation \( R \) be defined on \( \{2, 3, 5\} \) via \( (x, y) \in R \) if \( x < 2y \).
   (a) [3 pts] Draw the digraph for \( R \).
   (b) [4 pts] Is \( R \) reflexive? Explain or give a counterexample.
   (c) [4 pts] Is \( R \) symmetric? Explain or give a counterexample.
   (d) [4 pts] Is \( R \) transitive? Explain or give a counterexample.
   (e) [4 pts] Is \( R \) anti-symmetric? Explain or give a counterexample.
   (f) [4 pts] Is \( R \) a function? Justify your answer.

3. The relation \( xRy \) if \( x^2 = y^2 \) is an equivalence relation on the set \( \{-5, -1, 0, 1, 3, 5\} \).
   (a) [3 pts] List the members of each distinct equivalence class for \( R \).
   (b) [5 pts] In 1-2 sentences, explain what it means to say that your equivalence classes above partition the original set \( \{-5, -1, 0, 1, 3, 5\} \).

4. (a) [2 pts] Make up your own list of ordered pairs that describes a function \( f \) from the set \( \{a, b, c, d\} \) to the set \( \{3, 5, 7\} \).
   (b) [5 pts] Is your function onto? Explain.
   (c) [5 pts] Is your function one-to-one? Explain.
   (d) [2 pts] List the ordered pairs in the composition \( g \circ f \) of your function \( f \) and the function \( g(x) = x + 3 \).

5. [6 pts] Use the Euclidean algorithm to find the greatest common divisor of 81 and 159.

6. [6 pts] Completely evaluate \( 5^{129} \mod 12 \). (Yes, really.)

7. (a) [3 pts] Are there any Fibonacci numbers between 40 and 49? Justify your answer.
   (b) [10 pts] Prove that the following is true for the Fibonacci sequence \( \{f_n\} \):
   \[
   f_{n+1}^2 - f_{n-2}^2 = 4f_n f_{n-1}
   \]

   Continued on back
8. **[8 pts]** What will the following pseudo-code print if we begin with the command EXAM(1,5)?

```plaintext
EXAM(A,B)
IF 2A < B
  [PRINT A
   A:=2A
   B:=B+1
   EXAM(A,B)]
ELSE
  [PRINT B
   PRINT "Recursion is great."]
END
```

9. **In-Class Extra Credit [4 pts]:** Make up a formula that doesn’t involve $s_n$ for the sequence $\{\Omega_k\}$ of Problem #1d.

10. **Take-Home Problem [10 pts](Due Monday):** If $X$ and $Y$ are sets and $C$ represents the characteristic function, prove that

$$
C_{X\cup Y}(x) = \left\lfloor \frac{C_X(x) + C_Y(x)}{2} \right\rfloor - \left\lfloor \frac{C_X(x) \cdot C_Y(x)}{2} \right\rfloor.
$$

(The notation indicates the usual floor and ceiling functions that we studied.)