Math 131-01 - Dr. Miller - Exam #2 - 11/2/01

Work all problems in the blue book provided. Take this sheet with you when you leave.

1. Let $s_n = (-1)^n \cdot (2n-1)$ define a sequence.
   (a) *[3 pts]* List the first 5 terms of this sequence, beginning with s_1.
   (b) *[3 pts]* Is the sequence $\{s_n\}$ increasing, decreasing, or neither? Explain.
   (c) *[2 pts]* Evaluate $\prod_{n=3}^{5} s_{n-1}$.
   (d) *[4 pts]* Define $\Omega_k = \sum_{n=1}^{k} s_n$. Compute the values of Ω_3 and Ω_4.
2. Let a relation R be defined on $\{2, 3, 5\}$ via $(x, y) \in R$ if $x < 2y$.
   (a) *[3 pts]* Draw the digraph for R.
   (b) *[4 pts]* Is R reflexive? Explain or give a counterexample.
   (c) *[4 pts]* Is R symmetric? Explain or give a counterexample.
   (d) *[4 pts]* Is R transitive? Explain or give a counterexample.
   (e) *[4 pts]* Is R anti-symmetric? Explain or give a counterexample.
   (f) *[4 pts]* Is R a function? Justify your answer.
3. The relation xRy if $x^2 = y^2$ is an equivalence relation on the set $\{-5, -1, 0, 1, 3, 5\}$.
   (a) *[3 pts]* List the members of each distinct equivalence class for R.
   (b) *[5 pts]* In 1-2 sentences, explain what it means to say that your equivalence classes above partition the original set $\{-5, -1, 0, 1, 3, 5\}$.
4. (a) *[2 pts]* Make up your own list of ordered pairs that describes a function f from the set $\{a, b, c, d\}$ to the set $\{3, 5, 7\}$.
   (b) *[5 pts]* Is your function onto? Explain.
   (c) *[5 pts]* Is your function one-to-one? Explain.
   (d) *[2 pts]* List the ordered pairs in the composition $g \circ f$ of your function f and the function $g(x) = x + 3$.
5. *[6 pts]* Use the Euclidean algorithm to find the greatest common divisor of 81 and 159.
6. *[6 pts]* Completely evaluate 5^{129} mod 12. (Yes, really.)
7. (a) *[3 pts]* Are there any Fibonacci numbers between 40 and 49? Justify your answer.
   (b) *[10 pts]* Prove that the following is true for the Fibonacci sequence $\{f_n\}$:

$$f_{n+1}^2 - f_{n-2}^2 = 4 f_n f_{n-1}$$

Continued on back

8. *[8 pts]* What will the following pseudo-code print if we begin with the command EXAM(1,5)?

```
EXAM(A,B)
IF 2A < B
   [PRINT A
   A:=2A
   B:=B+1
   EXAM(A,B)]
ELSE
   [PRINT B
   PRINT "Recursion is great."]
END
```

9. *In-Class Extra Credit [4 pts]:* Make up a formula that doesn't involve s_n for the sequence $\{\Omega_k\}$ of Problem #1d.

10. *Take-Home Problem [10 pts](Due Monday):* If X and Y are sets and C represents the characteristic function, prove that

$$C_{X\cup Y}(x) = \left|\frac{C_X(x) + C_Y(x)}{2}\right| - \left|\frac{C_X(x)\cdot C_Y(x)}{2}\right|.$$

(The notation indicates the usual floor and ceiling functions that we studied.)