Math 131- Dr. Miller - Exam #2, Version #1 - 3/23/05

1. *[30 pts - 15 each]* Prove the following statements using mathematical induction:

   (a) $5^n - 1$ is divisible by 4 for all $n \geq 1$

   (b) For any integer $n \geq 2$, $\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n} > \frac{13}{24}$.

2. *[6 pts - 3 each]* Give an initial condition and a recurrence relation defining each sequence below:

   (a) $3, 6, 12, 24, \ldots$

   (b) $2, 6, 12, 20, \ldots$

3. *[10 pts - 5 each]* Now find the "solution," that is, the non-recursive formula for the nth term, of each sequence in Problem #2 above. Show work as needed.

4. *[6 pts]* List the first five terms of the recursive sequence defined as follows:

$$a_1 = 2, \quad a_2 = 3, \quad a_n = \begin{cases} (a_{n-1} + a_{n-2})/2 & \text{if } a_{n-1} + a_{n-2} \text{ is even} \\ 2a_{n-1} & \text{otherwise} \end{cases}$$

5. (a) *[5 pts]* A sequence is geometric with $a_2 = 3000$ and $a_3 = 3600$. Find a_1.

   (b) *[5 pts]* Does 9331 belong to the sequence $233, 240, 247, \ldots$? Explain (verbally).

6. *[8 pts]* Compute the following sum by any meaningful method, showing work.

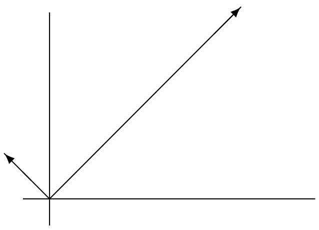
$$5 + 11 + 17 + \ldots + 6377$$

continued on back

7. *[5 pts]* State the formal mathematical definition of "function."

8. *[8 pts - 4 each]* Determine whether each of the following is a function with the indicated domain and codomain, clearly justifying your responses.

   (a) $f : \mathbf{R} \to \mathbf{R}$

   (b) $f : \{1, 2, 3, 4\} \to \mathbf{N}$ via $f = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$

9. *[8 pts - 4 each]* Determine whether each function is one-to-one with the indicated domain and codomain, clearly justifying your responses.

   (a) $f : \mathbf{R} \to \mathbf{R}$ via $f(x) = |x|$

   (b) $f : \mathbf{N} \to \mathbf{Z}$ via $f = \{(1, 99), (2, 98), (3, 97), (4, 96), \ldots\}$

10. *[9 pts - 3 each]* Let A denote the set of letters in the English alphabet. Consider the functions defined as follows:

$$f : \mathbf{N} \to A \quad \text{via} \quad f = \{(1, m), (2, a), (3, t), (4, h), (5, m), (6, a), (7, t), (8, h), \ldots\}$$

$$g : A \to A \quad \text{via} \quad g(x) = \begin{cases} x & \text{if } x \text{ is a vowel} \\ \text{the first vowel following } x & \text{otherwise} \end{cases}$$

$$h : \mathbf{N} \to \mathbf{N} \quad \text{via} \quad h(x) = x + 2$$

Find the following values, if possible; if not, tell why.

   (a) $g \circ f(3)$

   (b) $h \circ h(3)$

   (c) $h \circ f(3)$