

1. In each question below, simplify your answer, showing arithmetic work:
  - (a) [4 pts] I have 5 different stuffed toys at home and 3 nephews to share them with. In how many ways can I give each nephew one toy to play with?
  - (b) [4 pts] I have 6 different blankets that I bought on sale to give to charity. In how many ways can I select 2 of them to take to Goodwill?
  
2. In each question below, leave your answer in symbolic form:
  - (a) [4 pts] How many different 3-letter “words” can be formed using the letters VERMONT if you may not repeat any letter?
  - (b) [4 pts] How such “words” can be formed if you can repeat letters?
  - (c) [4 pts] How many different 3-person Executive Committees can be chosen in a club having 20 members?
  
3. In each question below, leave your answer in symbolic form.
  - (a) [8 pts] I want to play a card game in which I’ll be dealt 10 cards. In how many ways could I get 5 cards from each of 2 suits?
  - (b) [8 pts] In how many ways could I get 4 cards of one value, 3 of another, 2 of yet another, and 1 of a different value yet?
  
4.
  - (a) [5 pts] What is the coefficient of  $x^{12}y^{13}$  in the product  $(3x - y)^{25}$ ?
  - (b) [5 pts] What is the coefficient of  $a^4b^5c^2$  in the product  $(a + b + c)^3(a + b)^8$ ?
  - (c) [4 pts] Write the Binomial Theorem.
  
5.
  - (a) [4 pts] Draw an example of a graph having 5 vertices, one of which is an isolated vertex, and 5 edges, one pair of which are parallel edges. Identify the isolated vertex and the parallel edges in your diagram.
  - (b) [2 pts] Identify a pair of adjacent vertices in your graph.
  - (c) [4 pts] Is your graph above an example of a complete graph? Explain.
  - (d) [2 pts] Label each vertex with its degree.

*continued on back*

6. Consider the following graph for this question:

- (a) [4 pts] Is the graph bipartite? Justify your answer, referring to the definition of this concept in your explanation.
- (b) [6 pts] What is an Euler cycle, and does this graph contain one? Justify your answer by either listing the vertices for such a cycle or telling why this is impossible.

7. Of the two graphs below, one contains a Hamiltonian cycle and the other does not.

- (a) [3 pts] What is a Hamiltonian cycle?
- (b) [7 pts] Indicate which graph *does* contain one, then list the order in which the vertices would be traversed in such a cycle.
- (c) [7 pts] Explain why the other graph does not contain such a cycle.

8. *Take-Home Problems:*

- (a) [6 pts] Use the Binomial Theorem to prove that  $(-1)^n = \sum_{k=0}^n \binom{n}{k} (-2)^k$ . Show clear work, and explain your technique in a brief sentence.
- (b) [6 pts] *Clearly* illustrate the computations in using Dijkstra's Algorithm to find the shortest path from a to f. Be sure to report your final answer.