Math 131 - Dr. Miller - Final Exam, Version #1 - 5/4/05

All cell phones and watch alarms must be turned OFF.

1. *[3 pts]* Write the contrapositive of "If $x > y$ and y is positive, then $x^2 > y^2$."

2. *[15 pts - 2,3,5 each]* Negate each statement below, writing your answer in the same format (i.e., a verbal negation for verbal statements, a symbolic one for symbolic statements).

   (a) Every hexagon has 6 sides.

   (b) If x is rational and y is irrational, then $x + y$ is irrational.

   (c) $\forall x, y \in \mathbf{N},\ (x > y \quad \rightarrow \quad x^2 \geq y^2 + 1)$

   (d) $\forall x \in \mathbf{R},\ \exists n \in \mathbf{Z},\ (x \geq n \quad \wedge \quad x - n < 1)$

3. *[4 pts]* Find the truth value of the statement "$(q \vee \neg r) \rightarrow ((\neg p \wedge s) \rightarrow t)$" when $p \wedge q$ is true, $r \rightarrow s$ is false, and t is true.

4. *[6 pts]* Determine whether "$(p \vee q) \rightarrow \neg q$" and "$p \vee (\neg q \rightarrow q)$" are logically equivalent; show clear work and state your conclusion, but you need not explain.

5. *[5 pts - 1 each]* Let $S = \{1, \{2, 3\}, \{4\}\}$. Classify each statement below as true or false; you need not explain.

   (a) $1 \in S$

   (b) $\{2\} \subseteq S$

   (c) $\{2\} \in \mathcal{P}(S)$

   (d) $\{\{1\}\} \subseteq \mathcal{P}(S)$

   (e) $\{\{4\}\} \subseteq \mathcal{P}(S)$

6. *[4 pts]* Let $U = \{1, 2, 3, \ldots, 8\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{8, 7\}$ and $C = \{3, 5, 7\}$. Using correct notation, find $(A \cap B) \cup \overline{C}$.

7. *[5 pts]* In a 3-set Venn diagram, shade $A \cap \overline{(\overline{B} \cup C)}$ (note the double complement). Clearly show your final answer in a separate diagram.

8. *[8 pts - 3, 5 each]* Let the index set $I = \mathbf{N}$ and let $A_i = [0, 1/i) \cup \{i\}$.

   (a) Find $\cap_{i=2}^{4} A_i$.

   (b) Find $\cup_{i=1}^{\infty} A_i$.

9. *[8 pts]* Prove that $\sum_{i=1}^{n} (2i - 1) = n^2$ for all $n \in \mathbf{N}$.

10. *[8 pts]* Prove that $8^n - 3^n$ is divisible by 5 for all integers $n \geq 0$.

11. *[2 pts]* List the next 4 terms of the recursive sequence given by

$$a_1 = 1, \quad a_2 = 2, \quad a_n = \begin{cases} a_{n-1} + 1 & \text{if } a_{n-1} \text{ is odd} \\ a_{n-1} + a_{n-2} & \text{otherwise} \end{cases}$$

12. *[8 pts - 4 each]* Consider the sequence $2, 5, 10, 17, \ldots$.
    (a) Find both an initial condition and a recurrence relation for this sequence.
    (b) Find a solution for a_n (that is, a non-recursive formula). Show work, but you need not explain.

13. *[6 pts]* Use induction to prove that $a_n = n(n+1)$ is a solution for the sequence given by $a_1 = 2$ and $a_n = a_{n-1} + 2n$.

14. *[5 pts]* Use any meaningful technique to compute $81 + 87 + 93 + \ldots + 1227$. Show clear work.

15. *[6 pts - 3 each]* Tell whether each set of ordered pairs below is a function with domain $\mathbf{N}$ or not. Explain your responses.
    (a) $\{(2,4), (3,5), (4,6), \ldots\}$
    (b) $\{(1,2), (1,3), (1,4), \ldots\}$

16. *[6 pts - 3 each]* Tell whether each function below, with given domain and co-domain, is one-to-one or not. Explain your responses.
    (a) $f : \mathbf{Z} \longrightarrow \mathbf{N}$ via $f(x) = |x|$
    (b) $h : \mathbf{R} \longrightarrow \mathbf{R}^+$ via $h(x) = 2x$

17. *[6 pts - 3 each]* Now tell whether each function from Problem 16 above is onto or not. Explain your responses.

18. *[3 pts]* Use the Binomial Theorem to find the numeric coefficient of x^{15} in $(3x - 4)^{25}$.

19. *[12 pts - 4 each]* How many 5-card hands, drawn from a regular deck of cards have...
    (a) exactly 3 aces?
    (b) exactly 3 cards of the same value?
    (c) either 3 or 4 face cards? (The face cards are the jacks, queens, and kings.)

20. *[16 pts - 4 each]* My nephew is making a secret code-word by rearranging the letters of his name: TRAVIS.

    (a) How many have the letters AT (in that order) or the letters VIS (also in that order) first?

    (b) How many code-words can he make that will have both vowels together?

    (c) How many code-words have the vowels separated by at least one other letter?

    (d) How many code-words have a vowel on each end?

21. *[6 pts - 3 each]* For each degree sequence given below, either draw a *graph* that has that degree sequence, or tell why that is not possible.

    (a) 3,3,2,2,1

    (b) 3,3,2,2,0

22. *[6 pts]* Draw the complete graph on 5 vertices, then explain how Euler's Proposition applies to it.

23. *[3 pts]* Is the graph given by $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$ connected? Explain.

24. *[5 pts]* Are the graphs below isomorphic? Explain.

25. *[5 pts]* Does the graph below have an Euler trail? Specify one by listing the vertices you traverse in order, or explain why this is not possible.