1. [3 pts] Write the contrapositive of “If \( x > y \) and \( y \) is positive, then \( x^2 > y^2 \).”

2. [15 pts - 2,3,5 each] Negate each statement below, writing your answer in the same format (i.e., a verbal negation for verbal statements, a symbolic one for symbolic statements).
   
   (a) Every hexagon has 6 sides.
   (b) If \( x \) is rational and \( y \) is irrational, then \( x + y \) is irrational.
   (c) \( \forall x, y \in \mathbb{N}, (x > y \rightarrow x^2 \geq y^2 + 1) \)
   (d) \( \forall x \in \mathbb{R}, \exists n \in \mathbb{Z}, (x \geq n \land x - n < 1) \)

3. [4 pts] Find the truth value of the statement “\( (q \lor \neg r) \rightarrow ((\neg p \land s) \rightarrow t) \)” when \( p \land q \) is true, \( r \rightarrow s \) is false, and \( t \) is true.

4. [6 pts] Determine whether “\( (p \lor q) \rightarrow \neg q \)” and “\( p \lor (\neg q \rightarrow q) \)” are logically equivalent; show clear work and state your conclusion, but you need not explain.

5. [5 pts - 1 each] Let \( S = \{1, \{2,3\}, \{4\}\} \). Classify each statement below as true or false; you need not explain.
   
   (a) \( 1 \in S \)
   (b) \( \{2\} \subseteq S \)
   (c) \( \{2\} \in \mathcal{P}(S) \)
   (d) \( \{\{1\}\} \subseteq \mathcal{P}(S) \)
   (e) \( \{\{4\}\} \subseteq \mathcal{P}(S) \)

6. [4 pts] Let \( U = \{1, 2, 3, \ldots, 8\}, A = \{1, 2, 3, 4, 5\}, B = \{8, 7\} \) and \( C = \{3, 5, 7\} \). Using correct notation, find \( (A \cap B) \cup C \).

7. [5 pts] In a 3-set Venn diagram, shade \( A \cap (\overline{B \cup C}) \) (note the double complement). Clearly show your final answer in a separate diagram.

8. [8 pts - 3, 5 each] Let the index set \( I = \mathbb{N} \) and let \( A_i = [0, 1/i) \cup \{i\} \).
   
   (a) Find \( \bigcap_{i=2}^{4} A_i \).
   (b) Find \( \bigcup_{i=1}^{\infty} A_i \).

9. [8 pts] Prove that \( \sum_{i=1}^{n}(2i - 1) = n^2 \) for all \( n \in \mathbb{N} \).
10. [8 pts] Prove that $8^n - 3^n$ is divisible by 5 for all integers $n \geq 0$.

11. [2 pts] List the next 4 terms of the recursive sequence given by

\[ a_1 = 1, \quad a_2 = 2, \quad a_n = \begin{cases} 
  a_{n-1} + 1 & \text{if } a_{n-1} \text{ is odd} \\
  a_{n-1} + a_{n-2} & \text{otherwise}
\end{cases} \]

12. [8 pts - 4 each] Consider the sequence 2, 5, 10, 17, . . .

(a) Find both an initial condition and a recurrence relation for this sequence.
(b) Find a solution for $a_n$ (that is, a non-recursive formula). Show work, but you need not explain.

13. [6 pts] Use induction to prove that $a_n = n(n + 1)$ is a solution for the sequence given by $a_1 = 2$ and $a_n = a_{n-1} + 2n$.

14. [5 pts] Use any meaningful technique to compute $81 + 87 + 93 + \ldots + 1227$. Show clear work.

15. [6 pts - 3 each] Tell whether each set of ordered pairs below is a function with domain \( \mathbb{N} \) or not. Explain your responses.

(a) \( \{(2, 4), (3, 5), (4, 6), \ldots\} \)
(b) \( \{(1, 2), (1, 3), (1, 4), \ldots\} \)

16. [6 pts - 3 each] Tell whether each function below, with given domain and co-domain, is one-to-one or not. Explain your responses.

(a) \( f : \mathbb{Z} \rightarrow \mathbb{N} \) via \( f(x) = |x| \)
(b) \( h : \mathbb{R} \rightarrow \mathbb{R}^+ \) via \( h(x) = 2x \)

17. [6 pts - 3 each] Now tell whether each function from Problem 16 above is onto or not. Explain your responses.

18. [3 pts] Use the Binomial Theorem to find the numeric coefficient of $x^{15}$ in $(3x - 4)^{25}$.

19. [12 pts - 4 each] How many 5-card hands, drawn from a regular deck of cards have...

(a) exactly 3 aces?
(b) exactly 3 cards of the same value?
(c) either 3 or 4 face cards? (The face cards are the jacks, queens, and kings.)
20. [16 pts - 4 each] My nephew is making a secret code-word by rearranging the letters of his name: TRAVIS.

(a) How many have the letters AT (in that order) or the letters VIS (also in that order) first?
(b) How many code-words can he make that will have both vowels together?
(c) How many code-words have the vowels separated by at least one other letter?
(d) How many code-words have a vowel on each end?

21. [6 pts - 3 each] For each degree sequence given below, either draw a graph that has that degree sequence, or tell why that is not possible.

(a) 3,3,2,2,1
(b) 3,3,2,2,0

22. [6 pts] Draw the complete graph on 5 vertices, then explain how Euler’s Proposition applies to it.

23. [3 pts] Is the graph given by \( V = \{v_1, v_2, v_3, v_4, v_5\} \) and \( E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\} \) connected? Explain.


25. [5 pts] Does the graph below have an Euler trail? Specify one by listing the vertices you traverse in order, or explain why this is not possible.