Math 131 - Dr. Miller - Topics List for Exam #2, Fall 2024 - Wednesday, Oct. 30, 2024

Study this list together with your notes, readings, graded and ungraded HW problems, and in-class handouts. Aim to master concepts and techniques for use in general; just memorizing familiar problems is unlikely to be successful. Many former students use these lists to make their own study guides that include solved examples and reminders/cautions.

Sets: Memorize notation $\mathbf{Z}$, $\mathbf{Z}^+$, $\mathbf{Z}^-$, $\mathbf{Q}$, $\mathbf{Q}^+$, $\mathbf{Q}^-$, $\mathbf{R}$, $\mathbf{R}^+$, $\mathbf{R}^-$, $\emptyset$; 0 is even but not positive nor negative.

1. Convert between roster, set-builder notation. Know that letter names, intervals also define sets.
   (a) Only rosters and set-builders use curly braces. Intervals and letter names do not.
2. Know that def'n of set equality affects correct roster notation: no repeats, order irrelevant.
3. For the notation $\in$, $\notin$, $\subseteq$, $\not\subseteq$, $=$, and term "proper subset":
   (a) Demonstrate, identify its correct use in true/false, fill-in, or making your own examples.
   (b) Understand such notation when it is used in context throughout the course.
4. Given a set S, find its power set $\mathcal{P}(S)$; know how many elements $\mathcal{P}(S)$ should have.
   (a) Be careful with notation when sets are themselves elements of another set like the power set.
5. Know the notations $|S|$ and $n(S)$ and term "cardinality" for the number of elements in a set S.
6. Given roster sets, set-builder sets, intervals, or common set names ($\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{R}$, etc.), find their $\cup$, $\cap$, $\setminus$, $\times$, or complement, including when 2-3 tasks are mixed, as in $(A \cup B) \cap (C \setminus A)$ or $(A \cap B) \times C^c$.
7. Understand the terms disjoint, non-disjoint.
8. Create sets whose $\cup$, $\cap$, $\setminus$, $\times$, complement have certain features. If impossible, explain. (See HW #15.)
9. Predict the cardinality of a given Cartesian product.
10. Find $\cup$, $\cap$, $\setminus$, complement of sets in families, including infinite $\cup$, $\cap$.
    (a) Be careful with intervals vs. rosters, especially if they are originally given as set-builders.
    (b) Take care with bracket versus parenthesis for the infinite union, intersection of a family of intervals. Draw number lines to help.
    (c) Convert to better notation if you have "interval" answers such as $(a, a]$, (a, a), or $[a, a]$.

Relations: - Possible formats: set of ordered pairs, arrow diagram, xRy list, digraph, verbal/notation rule

1. Formally state the definition of relation (see notes - this should be a formal logic statement), of reflexive, symmetric, transitive.
2. Given a relation in one format above, rewrite it in my choice of another format.
   (a) By verbal/notation rule, I mean descriptions such as "xRy if x is a factor of y."
   (b) Understand the concepts of factor, multiple, and notation $x|y$. 0 is a multiple of every integer and a factor of none!
3. Given a relation in any format, identify or create true/false xRy examples, ordered pairs; justify if asked.
4. Know the distinction between the phrases "relation from A to B" and "relation on A."
5. Relations may be defined from/to/on power sets/other non-number sets (see p.350 #5,6,9; p.359 #22,26,27).
6. Given a relation in any format, determine whether it is/is not: reflexive, symmetric, transitive. I'll likely ask you to justify also.
   (a) It's a good idea for justifications to follow our former rules for true/false "for all" statements.
   (b) Remember that if the hypothesis for symmetric/transitive is NEVER met in the first place, then the entire conditional statement is TRUE and the relation IS symmetric/transitive.
7. Create digraphs/sets of ordered pairs with given mix of being/not: reflexive, symmetric, transitive.
8. Explain the term "equivalence relation." Identify a given relation as equiv/not.
9. Given a relation R in any format, find the relation's inverse R^{-1}.

Functions: Know that a function is a special kind of relation. Understand notation $f : A \to B$.

1. Understand, convert these ways to define functions: formulas, ordered pairs, arrow diagrams, $f(x)$-lists.

(continued on back)

2. Identify the domain, range, and codomain of a given function. For formulas, these may be infinite.
3. Find image, "$f(\_\_)$," pre-image, $f^{-1}(\_)$, of given elements of domain/codomain.
   (a) You may use alternative language (such as "input," "value," etc.) as in the box on pp.294-295.
   (b) Be prepared for examples where the domain/codomain are not just finite sets of numbers. (See Examples 7.1.6 and 7.1.7 in the reading and #7, #11 from Exer. Set 7.1).
4. State formal definitions of: function, one-to-one, onto.
5. Identify whether a given example (OPs, arrows, $f(x)$-list) is a function, one-to-one, onto.
   (a) Justify if asked: you can use the language of under/over-used elements, so long as you identify them as "domain elements" or "codomain elements."
6. Make up examples (OPs, arrows, $f(x)$-list) that are/aren't: functions, one-to-one, onto. Justify if asked.
7. Make up a formula that is NOT a well-defined function. Justify if asked, as in #27-28 p.303.
8. Find the arrow diagram or list of ordered pairs for the inverse relation of a given function; tell whether that inverse is itself a function or not, and justify.
9. State what it means to pass the VLT or HLT, to fail them.
10. Informally explain how the VLT, HLT connect to ideas about functions, one-to-one.
11. Given two functions via OPs, arrows, formulas, or $f(x)$-lists, write their composition as a set of OPs.

You will have the entire class period to take the exam. When you finish, you may hand it in and leave.

Please sit only in the rows we used for Exam #1, with empty rows between.
(This enhances integrity and gives me better access to come answer your questions.)

Students with SRU-approved accommodations should speak with me and process ODS requests ASAP.
ODS should proctor your exam since our classroom and my schedule are not automatically free.

Make-up Policy:

1. Notify me immediately if you'll miss the exam.
2. Documentation will be required: get a doctor's note, accident report, newspaper notice, etc.
3. If I excuse your absence, the Exam #2 content on our cumulative Final Exam in December will also be your make-up exam.
4. (D2L will show an artificial 0 for any excused absence until the end of the course.)