Math 131 - Dr. Miller - Topics List for Exam #3 - Monday, Nov. 25, 2024

Study this list together with your notes, readings, graded and ungraded HW problems, and in-class handouts. Aim to master concepts and techniques for use in general; just memorizing familiar problems is unlikely to be successful. Many former students use these lists to make their own study guides that include solved examples and reminders/cautions.

Sequences: Basics - *Bring a calculator for use on the exam.*

1. List the first few terms of a sequence given by an explicit formula, as in 5.1 #1-4.
   (a) Remember that "first" begins with the EARLIEST index, which isn't always a 1.
2. Find an explicit formula for a given listed sequence, as in 5.1 #6-12.
   (a) Be able to use my choice of domain, or to select your own as allowed.
   (b) Use (-1) to a correct power when signs alternate.
   (c) For fractions, try looking at numerator and denominator separately.
3. Evaluate sums or products when given starting and ending values for the index.
   (a) Problems may use sigma/pi notation, or ellipsis/expanded notation. See p. 184: #13-23 and 28-31.
   (b) Beware overly simple situations like "beginning index = ending index" or bucket contains a constant.
4. Convert expanded notation to sigma or pi notation, as in p.183 #38-41.
5. Rewrite a given sigma or pi expression in expanded form, showing its first 3 and last 2 terms, including when the upper and lower limits are expressions, not just concrete numbers.
6. Evaluate expanded form sums and products, including when the indexing variable "stops short."
7. "Peel off" or re-attach the last term in sigma or pi notation, as in p.183 #32-37.
8. Study problems like p. 184 #13 where the sequence is not given by a formula.
9. Perform a given change of variables on a sigma or pi expression, as in p.183 #48-53.

Recursion: - Spell out the two components (IC, and RR with domain) needed for a recursive sequence.

1. Explain (in words) the distinction between how an explicit formula vs recurrence relation use variables.
2. List the first few terms of a sequence given by initial conditions and a recurrence relation, including RRs with more than one a-variable.
3. Remember: recursive sequences start with initial conditions, so include ICs when listing first few terms.
4. Confirm whether a given explicit formula satisfies a RR, checking LHS and RHS separately. Be sure to draw a yes/no conclusion at the end. ("No" is indeed a possibility, as on HW #26.)

Special Sequences: - Factorials, Arithmetic, Geometric, Fibonacci-type

1. Write the expanded form of a given number or variable factorial, such as:
   (a) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, showing all factors for numbers below 10
   (b) $84! = 84 \cdot 83 \cdot 82 \cdot \ldots \cdot 3 \cdot 2 \cdot 1$, showing 3 terms first and last with ellipsis between for numbers over 10
   (c) $(k+1)! = (k+1) \cdot k \cdot (k-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$, also showing 3 terms start and end with ellipsis between for variables
   (d) Memorize that 0! is directly defined as $0! = 1$ we can't expand 0! as a product like above.
2. Use expanded form - either written out or in your head - to tell whether statements using factorials are true or false, such as $2 \cdot 3! = 6!$ (false) or $(k+1) \cdot k! = (k+1)!$ (true).
3. Simplify factorial notation as in 5.1 #57-59. Remember NOT to distribute coefficients or exponents.
4. Informally describe (in a sentence) the key features of arithmetic vs geometric vs Fibonacci-type sequences.
5. Identify given sequences as arithmetic, geometric, Fibonacci-type, or no special kind.
6. Understand, use the terms *common difference*, *common ratio*, and variable abbreviations CD, CR, d, r.
7. Create the first few terms of a sequence that fits given conditions, as on HW #27, Problem #2.
8. Sequences may use positive or negative integers, fractions, decimals, including for CDs and CRs.
9. Memorize the explicit formulas and domain ($n \geq 0$) for arithmetic, geometric sequences.
10. Given a few terms of an arithmetic or geometric sequence, write its explicit formula, including domain.
11. Use explicit formulas to find distant terms in arithmetic, geometric sequences, as in HW #27, Prob. #3.

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Counting: - Be prepared for situations where the steps aren't simply "blanks" in a word or code.

1. Find simple probabilities and list simple sample spaces, as in Section 9.1 Problems #3-13.
2. Draw possibility trees for a given situation, especially trees that are not symmetrical.
   (a) When told a few starting conditions (as in 9.1 #1-3, 5), include those in the tree for clarity.
3. Answer questions about maps. Drawings may/may not be required - read instructions to find out.
4. Apply mixtures of Multiplication, Addition, Subtraction, P.I.E. Principles in counting problems.
5. Problems may state outright that they permit or don't permit repeated symbols, or you may have to determine that for yourself from the context.
6. It's helpful to include rectangle diagrams separating categories or opposites, as in lecture.
7. Be able to separate outcomes by cases (inspired by interpretations that involve "or").
8. Comprehend and solve questions using the phrases "at least/most," "exactly," "none."
9. Show organized work such as blanks or verbal descriptions of steps for possible partial credit.
10. Review lots of HW problems and examples - these tasks need much practice!
11. Tasks needing a "combination step" WILL NOT appear on this exam.
12. List some or all permutations/combinations of a desired size from a given set.
13. Identify a given scenario as a permutation situation, combination situation, or neither, as in Activity #11.
14. For permutation/combination situations, write the notation for the answer.
15. Write the formula relating $P(n, r)$ and $C(n, r)$ (or its equivalent notation) for given n, r, as in HW.
16. Evaluate a given $P(n, r)$ or $C(n, r)$ by hand, with n, r equal to numbers OR expressions.

You should bring a calculator for use on this exam.

You will have the entire class period to take the exam. When you finish, you may hand it in and leave.

Please sit only in the rows we used for Exam #1, with empty rows between.
(This enhances integrity and gives me better access to come answer your questions.)

Students with SRU-approved accommodations should speak with me and process ODS requests ASAP.
ODS should proctor your exam since our classroom and my schedule are not automatically free.

Make-up Policy:

1. Notify me immediately if you'll miss the exam.
2. Documentation will be required: get a doctor's note, accident report, newspaper notice, etc.
3. If I excuse your absence, the Exam #3 content on our cumulative Final Exam in December will also be your make-up exam.
4. (D2L will show an artificial 0 for any excused absence until the end of the course.)