Math 131 - Dr. Miller - Topics List for Cumulative Final Exam, Fall 2024 - **Friday, Dec. 13, 8-10 am**

The exam is in our usual classroom. It is cumulative across the entire semester. Material is weighted roughly 10-15% on our latest work not covered on Exam #3, and the rest roughly equally spread across our previous three exam topics. Study the topics below, in conjunction with all materials from the course. Studying together is also a plus.

Logic with STATEMENT FORMS: (that is, using ONLY p, q, r, etc. and the symbols below)

1. Memorize and understand the symbols $\vee$, $\wedge$, $\sim$, $\rightarrow$, $\leftrightarrow$. Know, apply the logic order of operations.
2. Find isolated truth values for given statement forms, including when certain truth values hold for other statement forms, as in p.48, #21. Complete truth tables will NOT be asked on the final.
3. Determine whether statement forms are: *logically equivalent*, *tautology*, *contradiction*.
4. Remember the statement variables **t** and **c** for tautologies and contradictions.
5. MEMORIZE, WRITE key equivalences: de Morgan's Law, iff (mid p.45), stmt$\equiv$ctp, stmt$\not\equiv$ converse (p.48 #24,27).
6. Convert between verbal statements and statement FORMS as in p.36 #6-8, #10, p.48 #18. Prepare for conditionals and biconditionals as well.

Logic with STATEMENTS and PREDICATES: (i.e., using words, algebra, and/or $P(x)$ predicate notation)

1. Convert BETWEEN formal/informal; FROM symbolic TO verbal. (See p.48 #18 and Tarski's World.)
2. Fill in blanks to complete a rephrasing of a given statement, as in Exercise Sets 1.1, 3.1, and 3.2.
3. Recognize conjunctions, disjunctions, including the many verbal synonyms (such as "neither...nor"). Negate them.
4. Recognize verbal synonyms for conditional, biconditional statements, universal, existential.
5. Use, understand $\vee$, $\wedge$, $\sim$, $\rightarrow$, $\leftrightarrow$, $\forall$, $\exists$, plus basic symbols such as $=$, $<$, $\in$, $\subseteq$, etc.
6. Rewrite a biconditional statement as the conjunction of two conditionals.
7. Negate inequalities ($<$, $>$, $\leq$, $\leq$). Beware creating "illegal" 3-way inequalities.
8. Work with conditional rephrasings: if-then, only if, necessary, sufficient, implies, "trailing if":
   (a) Rewrite a statement using one of these synonyms so that it will use my choice of another.
   (b) Negate a statement that uses any of these synonyms.
   (c) Write the converse, inverse, or contrapositive of a statement that uses any of the synonyms.
   (d) Write converse, inverse, or contrapositive so that it too will use my choice of the synonyms.
9. Negate statements, including and, or, conditional, biconditional, universal, existential. Simplify when possible:
   (a) Apply de Morgan's Laws: for instance, "not both" isn't allowed; "not a or b"$\neq$"not (a or b)," etc.
   (b) Avoid double negatives: for instance, "not non-negative" isn't allowed. Negate inequalities correctly.
   (c) Don't "opposite" by mistake: "not positive"$\neq$"negative," negating $>$ doesn't create $<$, etc.
   (d) Also take care with inequality *words*, such as "at least/most," "over/under," etc.
   (e) Incorporate "not" into stmts/predicates: "It's not the case that," "Not all" and "there isn't/doesn't exist" are not allowed. "No" and "none" are also not allowed.
10. Understand/work with truth values of quantified statements, including multiple quantifiers, or blended such as "for all $x, y \in \mathbf{R}$."
    (a) Formally or informally justify true/false for a given quantified statement. FORMAL justifications need:
        i. True $\exists$s and false $\forall$s need clear, complete examples/counterexamples
        ii. False $\exists$s and true $\forall$s need you to explain how you exhausted all the possibilities.
    (b) Settings may be Tarski's World, a chart/list like Example 3.3.3, or just familiar sets of numbers.

Sets: Memorize notation $\mathbf{Z}$, $\mathbf{Z}^+$, $\mathbf{Z}^-$, $\mathbf{Q}$, $\mathbf{Q}^+$, $\mathbf{Q}^-$, $\mathbf{R}$, $\mathbf{R}^+$, $\mathbf{R}^-$, $\emptyset$; 0 is even but not positive nor negative; 1 is not prime.

1. Convert between roster, set-builder notation. Know that letter names, intervals also define sets.
   (a) Only rosters and set-builders use curly braces. Intervals and letter names do not.
2. Know that def'n of set equality affects correct roster notation: no repeats allowed/counted, order irrelevant.
3. Demonstrate, identify, understand correct use of $\in$, $\notin$, $\subseteq$, $\not\subseteq$, $=$, and term "proper subset."
4. Given a set S, find its power set $\mathcal{P}(S)$; know how many elements $\mathcal{P}(S)$ should have.
5. Know notations $|S|$, $n(S)$, and term *cardinality.* Predict the cardinality of a given Cartesian product.
6. Given roster sets, set-builder sets, intervals, or common set names ($\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{R}$, etc.), find their $\cup$, $\cap$, $\setminus$, $\times$, or complement, including when 2-3 tasks are mixed, as in $(A \cup B) \cap (C \setminus A)$ or $(A \cap B) \times C^c$.
7. Create sets whose $\cup$, $\cap$, $\setminus$, $\times$, complement have certain features, including *disjoint.* If impossible, explain.
8. Find $\cup$, $\cap$, $\setminus$, complement of sets in families, including infinite $\cup$, $\cap$.
   (a) Beware intervals vs. rosters, especially if originally given as set-builders. Draw number lines to help.
   (b) Take care with bracket versus parenthesis for the infinite union, intersection of a family of intervals.
   (c) Convert to better notation if you have "interval" answers such as $(a, a]$, (a, a), or $[a, a]$.

Relations: - Possible formats: set of ordered pairs, arrow diagram, xRy list, digraph, verbal/notation rule

1. Given a relation in one format above, rewrite it in my choice of another format.
   (a) By verbal/notation rule, I mean descriptions such as "xRy if x is a factor of y."
   (b) Understand terms *factor, multiple, prime*.0's a multiple of every integer but a factor of none!
2. Given a relation in any format, identify or create true/false xRy examples, ordered pairs; justify if asked.
3. Know the distinction between the phrases "relation from A to B" and "relation on A."
4. Relations may be defined from/to/on power sets/other non-number sets (see p.350 #5,6,9; p.359 #22,26,27).
5. Given a relation in any format, determine whether it is/is not: reflexive, symmetric, transitive. Justify informally.
   (a) Remember that if the hypothesis for symmetric/transitive is NEVER met in the first place, then the entire conditional statement is TRUE and the relation IS symmetric/transitive.
6. Create digraphs/sets of ordered pairs with given mix of being/not: reflexive, symmetric, transitive.

Functions: Know that a function is a special kind of relation. Understand notation $f : A \to B$.

1. Understand, convert these ways to define functions: formulas, ordered pairs, arrow diagrams, $f(x)$-lists.
2. Identify the domain, range, and codomain of a given function. For formulas, these may be infinite.
   (a) Be prepared for examples where the domain/codomain are not just finite sets of numbers. (See Examples 7.1.6 and 7.1.7 in the reading and #7, #11 from Exer. Set 7.1).
3. Identify whether a given example (OPs, arrows, $f(x)$-list) is a function, one-to-one, onto. Justify if asked.
   (a) You can discuss under/over-used elements, but identify them as "domain elements" or "codomain elements."
   (b) You may use alternative language (such as "input," "value," etc.) as in the box on pp.294-295.
4. Make up your own examples (OPs, arrows, $f(x)$-list) that are/aren't: functions, one-to-one, onto. Justify if asked.

Sequences: Basics, Recursion, Special - *You should bring a basic calculator for use on the exam.*

1. List the first few terms (meaning *earliest*, NOT subscript=1, nor only RR-created) of a sequence from an explicit formula or recursive definition.
2. Find an explicit formula for a given listed sequence, as in 5.1 #6-12, possibly using MY choice of domain.
3. Evaluate sums or products when given starting and ending values for the index.
   (a) Problems may use sigma/pi notation, or ellipsis/expanded notation. See p. 184: #13-23 and 28-31.
   (b) Beware overly simple situations like "beginning index = ending index" or bucket contains a constant.
4. Convert expanded notation to sigma or pi notation, as in p.183 #38-41.
5. Rewrite a given sigma or pi expression in expanded form, showing its first 3 and last 2 terms, including when the upper and lower limits are expressions, not just concrete numbers.
6. Evaluate expanded form sums and products, including when the indexing variable "stops short."
7. Perform a given change of variables on a sigma or pi expression, as in p.183 #48-53.
8. Write the expanded form of a given number or variable factorial. Use it to simplify notation as in 5.1 #57-59. Remember NOT to distribute coefficients or exponents.
9. Use expanded form - either written out or in your head - to tell whether statements using factorials are true or false, such as $2 \cdot 3! = 6!$ (false) or $(k+1) \cdot k! = (k+1)!$ (true).
10. Identify given sequences as arithmetic, geometric, Fibonacci-type, or no special kind.
11. Understand, use the terms *common difference*, *common ratio*, and variable abbreviations CD, CR, d, r.
12. Create the first few terms of a sequence that fits given conditions, as on HW #27, Problem #2.
13. Sequences may use positive or negative integers, fractions, decimals, including for CDs and CRs.
14. Memorize the explicit formulas and domain ($n \geq 0$) for arithmetic, geometric sequences.
15. Given a few terms of an arithmetic or geometric sequence, write its explicit formula, including domain.
16. Use explicit formulas to find distant terms in arithmetic, geometric sequences, as in HW #27, Prob. #3.

Counting: - Steps in a process aren't always "blanks," and can now include "combination steps."

1. Apply mixtures of Multiplication, Addition, Subtraction, P.I.E. Principles in counting problems.
   (a) Rules about repeated symbols may be stated outright, or you may have to determine that from the context.
   (b) It's helpful to include rectangle diagrams separating categories or opposites, as in lecture.
   (c) Consider whether choices are treated same or different to decide if you need a C(n,r) step.
2. Comprehend and solve questions using the phrases *exactly*, *none*, *at least/most*, including "at least 1."
3. Review lots of HW problems and examples - these tasks need much practice!
4. Memorizeformulas for $P(n,r)$, $C(n,r)$ (or equiv. notation)Evaluate with n, r equal to numbers OR expressions.
5. Use substitution to confirm a given identity involving $P(n,r)$ or $C(n,r)$ notation, as in HW.
6. The Binomial Theorem will NOT appear on the Final, but will have one last HW of its own.

Graph Theory: - This will NOT appear on the final, but will be assessed in last HW assignments.

You will have the full 2-hour period to take the exam. When finished, you may hand it in and leave.

Bring a calculator (non-phone) for the exam.