Key $\frac{20}{20}$ $\frac{\cancel{18}}{\cancel{18}}$

Math 131 - Dr. Miller - Exam #1 - Wednesday, Sept. 28, 2011

Address each problem carefully and thoroughly. This exam is worth 100 points.

1. (a) *[10 pts]* Are the statement forms $q \Rightarrow \sim (p \vee q)$ and $\sim p \vee \sim q$ logically equivalent? Justify your response. with

(-1) 1 or 2 errors
(-2) wrong rewrite of any stmt. "portion"
(-1) "No."
(-7) Verbal attempt w/ flawed rewrites.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \vee q$	$\sim(p \vee q)$	$q \Rightarrow \sim(p \vee q)$
T	T	F	F	F	T	F	F
T	F	F	T	T	T	F	T
F	T	T	F	T	T	F	F
F	F	T	T	T	F	T	T

These columns don't agree.
They are not logically equivalent.

(b) *[~~8~~ 10 pts]* Construct a truth table for the statement form $(p \wedge \sim q) \Rightarrow r$.

(-3) $\Rightarrow$ equals $\wedge$
(-1) 1 or 2 errors

p	q	r	$\sim q$	$p \wedge \sim q$	$(p \wedge \sim q) \Rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

5 mins.

$\frac{30}{30}$

2. [15 pts - 5 each] For this problem, consider the following statement variables:

p: x is prime $\qquad$ q: x is odd $\qquad$ r: $x > 7$

(-2) AND instead of OR, IMPLIES.

(a) Rewrite $(r \vee \sim p) \Rightarrow \ \sim q$ verbally.

If $x > 7$ or x is not prime, then x is not odd. (even)

(-2) each bad connective. neither-nor ok, but not preferred.

(b) Rewrite $q \wedge \sim (r \vee p)$ verbally.

x is odd and $x \leq 7$ and x is not prime.

(c) Rewrite "x is an even prime only if it's prime ~~and~~ or less than or equal to 7" ENTIRELY symbolically, using the given p, q, and r.

$$(p \wedge \sim q) \Rightarrow (p \vee \sim r)$$

(-5) overall hypoth/concl. bkwds.

15 - 5 ea.

3. Consider the statement "$\sin x > 0$ and $\cos x < 0$ (concl.) if the angle x is in Quadrant II (hypoth.)."

(-5) still conditional.
(-2) backwards.

(a) [5 pts] Rewrite the original statement as a universal statement.

For every angle x in Quadrant II, $\sin x > 0$ and $\cos x < 0$.

(-4) "If $x \in$ QII," then necess. makes QII hypoth. AND "and" is backwards
(-3) backwards
(-3) univ. or ctp.

(b) [5 pts] Write the converse of the original statement using the phrase "necessary."

Angle x being in Quadrant II is necessary for $\sin x$ to be positive and $\cos x$ negative.

(c) [5 pts] Write the inverse of the original statement using the word "sufficient."

Angle x not being in Quadrant II is sufficient for $\sin x \leq 0$ or $\cos x \geq 0$ to occur.

(-2) "and"] (-3) max
(-2) bkwds
(-1) $x \in$ QII
(-1) "if"

5 mins.

$\frac{35}{35}$

4. *[15 pts - 5 each]* Convert each verbal statement to totally symbolic form (no words) and vice versa.

(-3) $\exists x \in \mathbb{R}$, ①
(-1) wrong set anywhere
(-1) quantifiers
(-2) "but" = "or"
max ② quant/but error.

(a) Some real numbers are rational.

$\exists x \in \mathbb{R},\ x \in \mathbb{Q}$

(b) x is either a rational number but not an integer, or else $x = 0$.

$(x \in \mathbb{Q} \wedge x \notin \mathbb{Z}) \vee x = 0.$

(c) There is a real number x for which xy is rational for all rational numbers y.

$\exists x \in \mathbb{R},\ \forall y \in \mathbb{Q},\ xy \in \mathbb{Q}.$

5. *[20 pts - 5 each]* Write the negation of each statement below. You may use your choice of verbal or symbolic form.

(-3) didn't reverse quantifiers
(-2) missed 1 quant.
(-3) Some aren't.

(a) There is a real number x for which xy is rational for all rational numbers y.

For every real number x, there is a rational number y for which xy is not rational.

$\forall x \in \mathbb{R},\ \exists y \in \mathbb{Q},\ xy \notin \mathbb{Q}$

* (b) Some rectangles are squares.

No rectangles are squares.
All rectangles are not squares.

(-1) "or" left
(-2) kept "if-then"
(-1) $xy \neq 0$

(c) If $xy = 0$, then $x = 0$ or $y = 0$.

$xy = 0$ but (and) $x \neq 0$ and $y \neq 0$.

* (d) All pentagons are polygons. (AVOID the word "not" for this answer.)

BAD instructions: Full credit for every student.
Correct answer: Some pentagons are not polygons.

* 15 5

6. [~~18~~ 15 pts - ~~6~~ 5 each] Identify each statement below as true or false, then justify your claim using an appropriate method.

(-2) False w/ div. by 2 argument.
(-4) T w/ no argument.

(a) Some even integers are prime.

True -

Example: 2 is an even integer that is prime.

(-1) 51 as cex
(-2) include 49 in cex

(b) Any integer x where $48 \leq x \leq 51$ is not prime.

True

Exhaust domain: 48, 49, 50, and 51 are indeed not prime.

(-3) F ⇒ F is false

(c) If 6 is a prime number and 4 is not, then $6 + 4 = 15$.

True.

The hypothesis is false, so the overall conditional statement is true.