Math 131 - Dr. Miller - Answers to Exam #1, Version #1 (green)

1. (a) You can't have fries and you can't have salad. (The original was of the form "fries $\vee$ salad.")
   (b) You don't like pizza but/and you're not crazy.
   (c) Sometimes when it rains, my basement does not flood. (The original was of the form "$\forall$ rain, house floods.")
2. (a) If you're crazy, you don't like pizza.
   (b) If it's not a mammal, then it's not a cat. (OR Non-mammals are not cats.)
3. (a) $\forall$ squares S, S is a polygon.
   (b) $\forall x \geq y$, $x^2 \geq y^2$.
   (c) $\exists x \in \mathbf{R}$, $x^2 < x$. (OR $\exists$ real number x, $x^2 < x$.)
4. (a) $(p \vee q) \wedge r$ (hypothesis and not conclusion), which is $(p \wedge r) \vee (q \wedge r)$
   (b) $\exists x \in \mathbf{Q}, \exists y \in \mathbf{Z}, x > y$ (*only* the quantifier changes in the "quantifier phrase" part.
   (c) $\forall a \in R, a \leq 0$ or $a^2 \geq a$
5. Method 1: truth table

p	q	$p \vee q$	$\neg p$	$\neg p \to q$	$(p \vee q) \wedge (\neg p \to q)$	$\neg q$	$p \to \neg q$
t	t	t	f	t	t	f	f
t	f	t	f	t	t	t	t
f	t	t	t	t	t	f	t
f	f	f	t	f	f	f	t

$(p \vee q) \wedge (\neg p \to q)$ is not logically equivalent to $p \to \neg q$ because their columns in the truth table are not identical.

Method 2: algebra

$$\begin{aligned} (p \vee q) \wedge (\neg p \to q) &\Leftrightarrow (p \vee q) \wedge (p \vee q) \quad \text{(implication is "not hypoth, or concl")} \\ &\Leftrightarrow (p \vee q) \quad \text{("idempotence")} \end{aligned}$$

yet $p \to \neg q \Leftrightarrow \neg p \vee \neg q$, not the same.

6.

$$\begin{aligned} (p \to q) \vee \neg(p \wedge \neg q) &\Leftrightarrow (p \to q) \vee (\neg p \vee q) \quad \text{(negate the "and")} \\ &\Leftrightarrow (\neg p \vee q) \vee (\neg p \vee q) \quad \text{(implication is "not hypoth, or concl")} \\ &\Leftrightarrow \neg p \vee q \quad \text{("idempotence")} \end{aligned}$$

7. $r \wedge u$ true means that both r and u must be true. So we get

$$\begin{aligned} s \wedge (t \vee \neg r) \to \neg u \quad \text{" = "} &\quad \text{true } \wedge \text{ (false } \vee \neg \text{ true) } \to \neg \text{ true} \\ \text{" = "} &\quad \text{true } \wedge \text{ (false } \vee \text{ false) } \to \text{ false} \\ \text{" = "} &\quad \text{true } \wedge \text{ false } \to \text{ false} \\ \text{" = "} &\quad \text{false } \to \text{ false ,} \quad \text{which is true.} \end{aligned}$$

8. (a) true – $0 = -2 + 2$, a sum of integers whose squares are equal.
   (b) true – $4 = 2 + 2$, a sum of integers whose squares are equal.
   (c) false – if $x^2 = y^2$, x must equal $\pm y$, so that $x + y = 0$ or $2y$, making $y = 1/2$, not an integer.
   (d) true – the empty set is a subset of every set, so it's a member of every power set.
   (e) true – the empty set is a subset of every set.
   (f) false – a subset's elements always belong to the "bigger" set, but the element 1 by itself does not belong to $\mathcal{P}(\{1\})$

9. (a) $(A \cup C) \cap \overline{B} = \{6, 7, 9\} \cap \overline{B} = \{6, 7, 9\} \cap \{6, 8\} = \{6\}$
   (b) $C \setminus B = \{6, \not 7\} = \{6\}$
   (c) $C \times B = \{(6, 5), (6, 7), (6, 9), (7, 5), (7, 7), (7, 9)\}$

10.

11. (a) $A_3 = \{-1, 1, 3\}, A_4 = \{-1, 1, 4\},$, and $A_5 = \{-1, 1, 5\}$, so $\cap_{i=3}^{5} A_i = \{-1, 1\}$
   (b) $\cup_{i=3}^{5} A_i = \{-1, 1, 3, 4, 5\}$
   (c) $\cap_I A_i = \{-1, 1\}$

12. (a) $\cup_I A_i = \mathbf{R} = (-\infty, \infty)$
   (b) $\cap_I A_i = \{0\}$