Key

$\frac{25}{25}$

Math 131 - Dr. Miller - Exam #3 - Fall 2011

1. For all parts of this problem, let the universal set $U = \{21, 22, 23, \ldots, 30\}$, and let $A = \{x \in U \mid x = 2 \; mod \; 3\}$, $B = \{21, 23, 25, 27, 29\}$, $C = \{24, 27\}$, and $D = \{21, 25, 26, 30\}$.

(a) *[2 pts]* Rewrite A using correct listing notation.

$A = \{23, 26, 29\}$

-1 plural

(b) *[2 pts]* Rewrite B using correct set-builder notation.

$B = \{x \in U \mid x \text{ is odd}\}$ OR
$B = \{x \in \mathbb{Z} \mid x \text{ is odd and } 21 \leq x \leq 29\}$

(c) *[2 pts]* Use the $\notin$ symbol correctly in a symbolic statement about C.

(various) $23 \notin C$

-2 element

(d) *[2 pts]* Use the $\subseteq$ symbol correctly in a symbolic statement about C.

(various) $\{23\} \subseteq C$

-1 5^2

(e) *[2 pts]* How many elements does $\mathcal{P}(B)$ contain?

$2^5 = 32$ subsets $= 32$ elements of $\mathcal{P}(B)$

-2 "subset of $C \cup D$" =
-1 $\mathcal{P}(C \cup D)$

(f) *[5 pts]* Using correct notation, write a subset of $\mathcal{P}(C \cup D)$.

(various) elements of $\mathcal{P}(C \cup D)$: $\{21, 24, 25\}$, $\{30\}$

subset of $\mathcal{P}(C \cup D)$: $\{\{21, 24, 25\}, \{30\}\}$

-3 $\{24, 27\}$

(g) *[5 pts]* Using correct notation, find $C \setminus B$.

$C = \{24, \cancel{27}\}$

~~$B = \{21, 23, 25, 27, 29\}$~~

$C \setminus B = \{24\}$

-2 $\cap C$

(h) *[5 pts]* Using correct notation, find $(D^c \cap B) \cup C$.

$D^c = \{22, 23, 24, 27, 28, 29\}$

$D^c \cap B = \{23, 27, 29\}$

$(D^c \cap B) \cup C = \{23, 24, 27, 29\}$

33, F11

24/24

(-2) $P \cap X = \{2\}$
(-2) wrong size P

2. Use the set $X = \{2, 4, 6, 8\}$ for all parts of this problem.

(a) *[4 pts]* Make up a 5-element set P so that the cardinality of $P \cap X$ is 2. If not possible, explain why.

(Various) $P = \{2, 4, a, b, c\}$

(b) *[4 pts]* Make up a 4-element set Q so that $Q \setminus X = \{2, 5, 7\}$. If not possible, explain why.

Not possible – in discarding members of X, the number 2 would have to be discarded, not kept.

(c) *[4 pts]* Make up a set R so that $R \times X$ contains 12 elements (you need not list them). If not possible, explain why.

(Various) $R = \{a, b, c\}$

(-1) each bad paren or bracket
(-2) each wrong #

3. *[12 pts - 3 each]* Define $A_i = \left(-\frac{1}{i}, 2 - \frac{1}{i}\right]$ for all positive integers i.

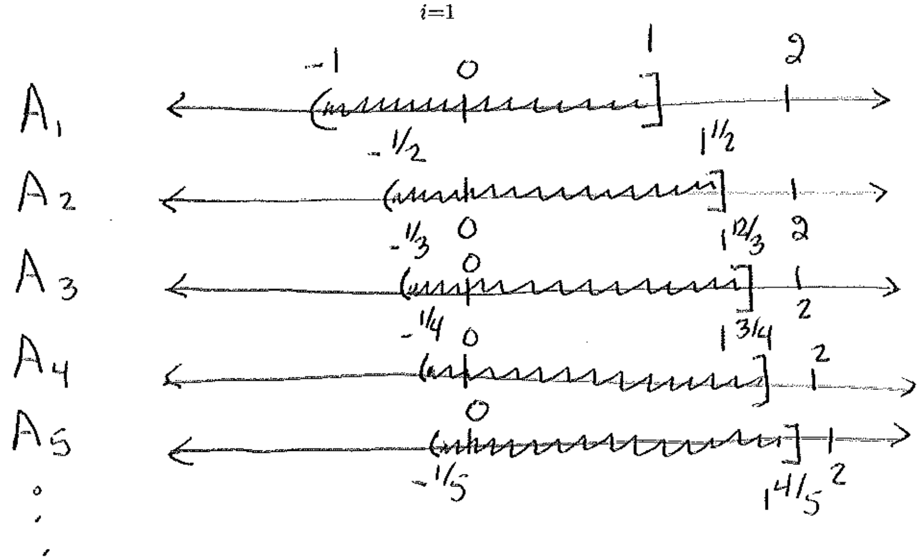
(a) Using correct notation, find $\bigcup_{i=3}^{5} A_i$. $= (-1/3, 1\,4/5]$

(b) Using correct notation, find $\bigcap_{i=3}^{5} A_i$. $= (-1/5, 1\,2/3]$

(-1)]

(c) Using correct notation, find $\bigcup_{i=1}^{\infty} A_i$. $= (-1, 2)$

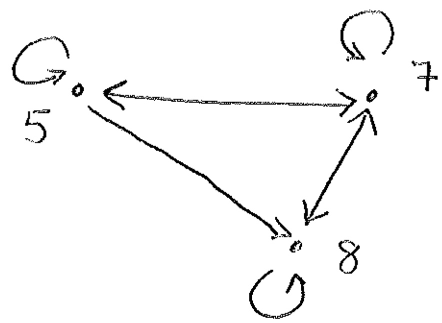
(d) Using correct notation, find $\bigcap_{i=1}^{\infty} A_i$. $= [0, 1]$



$\frac{26}{26}$

4. Consider the relation on $A = \{5, 7, 8\}$ defined by xRy if $x - y < 3$.

   (a) *[3 pts]* Draw the directed graph for R.



   (b) *[3 pts]* Using correct notation, list R as a set of ordered pairs.

   $R = \{(5,5), (5,7), (5,8), (7,5), (7,7), (7,8), (8,7), (8,8)\}$

   (c) *[5 pts]* Is R reflexive? If yes, explain; if no, give a counterexample.

   Yes - every vertex has a loop arrow.
   OR every number is related to itself.

(-3) example in support
(-1) irrelevant pairs.

   (d) *[5 pts]* Is R symmetric? If yes, explain; if no, give a counterexample.

   No - the arrow between 5 and 8 isn't double-headed.

   OR $5 R 8$ but $8 \not R 5$.

(-2) wrong "middle man"
(-3) example in support
(-2) reversed

   (e) *[5 pts]* Is R transitive? If yes, explain; if no, give a counterexample.

   No - $8 R 7$ and $7 R 5$ but $8 \not R 5$.

   (f) *[5 pts]* Is R a function? Explain.

   No - R repeats first coordinates
   OR 5 has more than one image.

5. *[15 pts - 5 each]* Consider the relation on $\mathbf{Z}$ given by xSy if $x + y > 2$.

(-1) irrelevant pairs

(a) Is S reflexive? If yes, explain; if no, give a counterexample.

No. ~~$5 \not S 5$~~

$0 \not S 0$

(-3) one example

(b) Is S symmetric? If yes, explain; if no, give a counterexample.

Yes – If $x + y > 2$,
then $y + x$ is still greater than 2.

(-2) botched hypoth or concl R vs $\not R$.

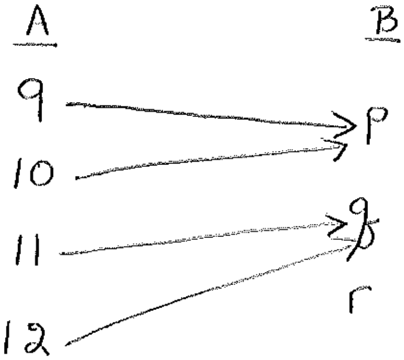
(c) Is S transitive? If yes, explain; if no, give a counterexample.

No. $1 S 3$ and $3 S 0$, but $1 \not S 0$.

6. (a) *[5 pts]* Make up an example of a function from $A = \{9, 10, 11, 12\}$ to $B = \{p, q, r\}$ that is not one-to-one. You need not explain.

(-2) not a function

(Various)



(-2) includes 1-1

(b) *[5 pts]* Is your example above onto? Explain.

(Various) Mine is not: $r \in B$ does not belong to the range/ isn't used as an image.