Math 131 - Dr. Miller - Exam #3, Version #1 - 4/23/05

Answers

1. (a) $C(7,3) = \frac{7!}{4!3!} = \frac{7 \cdot \not{6} \cdot 5}{\not{3} \cdot \not{2}} = 35$

   (b) $P(300,4) = \frac{300!}{296!} = 300 \cdot 299 \cdot 298 \cdot 297 = 7,938,988,200$

   (c) $C(6,0) = \frac{6!}{6!0!} = 1$

2. (a) $C(28,6)$ - He won't use them any differently.

   (b) $C(20,3)$ - They won't have different tasks.

   (c) $C(15,3)$ - They all go in the calzone.

   (d) $P(15,3)$ - They're treated differently: top stripe, middle, and bottom.

3. $C(7,0) \quad C(7,1) \quad C(7,2) \quad C(7,3) \quad C(7,4) \quad C(7,5) \quad C(7,6) \quad C(7,7)$

4. (a) Total expression is $C(9,3) \cdot (3x)^6 \cdot (-2y)^3$, so numeric coefficient is $-C(9,3) \cdot 3^6 \cdot 2^3$

   (b) $x^6 = (x^2)^{21-k} \cdot (\frac{1}{x})^k$, so $6 = 42 - 2k - k$, and $k = 12$. Total expression is then $C(21,12) \cdot (x^2)^9 \cdot (\frac{3}{x})^{12}$, and numeric coefficient is $C(21,12) \cdot 3^{12}$.

5. (a) $26^3 \cdot 10^2 \cdot 26 + 26^3 \cdot 10^3 \cdot 26$

   (b) $26^3 \cdot 9^2 \cdot 26 + 26^3 \cdot 9^3 \cdot 26$

   (c) part (a) minus part (b)

6. (a) $6 \cdot 6 \cdot 6$ - Pick the Pegs 1 and 4 color (6 choices), then Peg 2 (6 choices), then Peg 3 (6 choices).

   (b) $6 \cdot 5 \cdot 4$ - Pick the Pegs 1 and 4 color (6 choices), then Peg 2 (5 colors left), then Peg 3 (4 colors).

   (c) $C(4,2) \cdot 6 \cdot 5 \cdot 4$ - Pick where the matching pegs will go (they'll be colored the same, so order doesn't matter: $C(4,2)$), then pick the matching color (6 choices), then the earliest uncolored peg's color (5 colors left), then the last color (4 colors).

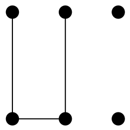
7. (a) $8 \cdot 5! \cdot 7!$ - Decide which seats are for the first graders (1-5, 2-6, ... out to 8-12 is 8 options), then seat the first graders in a row (5!), then seat the others (7!).

   (b) $3 \cdot 2 \cdot 10!$ - Seat the near-end third-grader (3 choices), then the far-end one (2 kids left), then seat all the others in a row (10!).

   (c) $7 \cdot 6 \cdot 10!$ - Seat someone on the near end (7 choices), someone on the far end (6 choices left), then seat all the others in a row (10!).

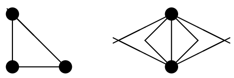
8. (a) $C(4,1) \cdot C(13,5)$ - Pick one suit from the four, then pick 5 cards from that suit.

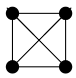
   (b) $C(4,1) \cdot C(13,2) \cdot C(13,1) \cdot C(13,1) \cdot C(13,1) \cdot C(13,1)$ - Pick the suit to be "doubled" ($C(4,1)$), then 4 cards from that suit ($C(13,2)$), then one each from each remaining suit. Note that $C(13,1) \cdot C(13,1) \cdot C(13,1) \cdot C(13,1) \cdot C(48,1)$ overcounts, since it "induces" order and makes it seem that the first card and the last card are treated differently.

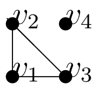
9. (a) 

   (b) Not possible since there are an odd number of odd-degree vertices

   (c) Not possible for a *graph* since none of the 5 vertices can connect to 5 *different* vertices. It is possible if we have a *pseudo-graph* (note that software cannot draw curved edges):

   (d) 

   (e) 

10. (a) The sum of the degrees of all vertices equals 2 times the number of edges.

    (b) At only degree 12 each, the vertices produce a degree sum of $25 \cdot 12 = 300$, yet 2 times the number of edges equals 304, so we need higher degrees somewhere.