Math 210-310 - Dr. Miller - Shared Homework #12-S: Non-Algebraic Proportion Methods

1. Use a scaling approach for solving these problems:
   (a) Allie bought half a dozen donuts for \$1.80. How much would 4 and a half dozen donuts cost?
   (b) On a map, $1\frac{1}{2}$ inch corresponds to 12 miles. Two cities are shown as 6 inches apart on the map. How far apart are they in reality?
   (c) Jimmy paid \$44 for 8 pizzas. How much will 20 pizzas cost?
   (d) It took Lorenz 8 hours to grade the first term papers for his 20 students. If 2 students drop the class, how long will it take him to grade the second round of term papers in that class?
   (e) My recipe calls for 5 cups of flour to 2 cups of sugar. How much flour do I need if I use 7 cups of sugar?
   (f) Two thirds of a cup of sugar is needed to make 6 dozen dainty candies. How many dozen candies can be made using 4 cups of sugar?
2. Now use a unit-rate approach for solving the same problems:
   (a) Allie bought half a dozen donuts for \$1.80. How much would 4 and a half dozen donuts cost?
   (b) On a map, $1\frac{1}{2}$ inch corresponds to 12 miles. Two cities are shown as 6 inches apart on the map. How far apart are they in reality?
   (c) Jimmy paid \$44 for 8 pizzas. How much will 20 pizzas cost?
   (d) It took Lorenz 8 hours to grade the first term papers for his 20 students. If 2 students drop the class, how long will it take him to grade the second round of term papers in that class?
   (e) My recipe calls for 5 cups of flour to 2 cups of sugar. How much flour do I need if I use 7 cups of sugar?
   (f) Two thirds of a cup of sugar is needed to make 6 dozen dainty candies. How many dozen candies can be made using 4 cups of sugar?

1. (a) Additive scaling:

$$\begin{array}{rcr} 1/2\ \textit{dozen} & : & \$\ 1.80 \\ 1\ \textit{dozen} & : & \$\ 3.60 \\ 1\ \textit{dozen} & : & \$\ 3.60 \\ 1\ \textit{dozen} & : & \$\ 3.60 \\ +\ 1\ \textit{dozen} & : & +\ \$\ 3.60 \\ \hline 4\frac{1}{2}\ \textit{dozen} & \textit{costs} & \boxed{\$\ 16.20} \end{array}$$

Direct scaling: (Scale factor is $4\frac{1}{2} \div \frac{1}{2} = 9$.)

$$\begin{array}{rcr} 1/2\ \textit{dozen} & : & \$\ 1.80 \\ \downarrow\ \times 9 & & \times 9\ \downarrow \\ 4\frac{1}{2}\ \textit{dozen} & \textit{costs} & \boxed{\$\ 16.20} \end{array}$$

(b) Additive scaling: MANY arrangements of scratchwork are possible. Here's one:

$1\frac{1}{2}$ inches equals 12 miles, so 3 inches equals 24 miles:

$$\begin{array}{rcr} 3\ \textit{inches} & & 24\ \textit{miles} \\ +\ 3\ \textit{inches} & : & +\ 24\ \textit{miles} \\ \hline 6\ \textit{inches} & & \boxed{48\ \text{miles}} \end{array}$$

Direct scaling: (Scale factor is $6 \div 1\frac{1}{2} = 4$.)

$$\begin{array}{rcr} 1\frac{1}{2}\ \textit{inches} & : & 12\ \textit{miles} \\ \downarrow\ \times 4 & & \times 4\ \downarrow \\ 6\ \textit{inches} & : & \boxed{48\ \text{miles}} \end{array}$$

(c) Additive scaling: MANY arrangements of scratchwork are possible. Here's one:

$$\begin{array}{rr} \$44 & 8\ \textit{pizzas} \\ \$44 & 8\ \textit{pizzas} \\ +\ \$22 & +\ 4\ \textit{pizzas} \\ \hline \boxed{\$\ 110} & \boxed{20\ \text{pizzas}} \end{array}$$

Direct scaling: (Scale factor is $20 \div 8 = 2.5$.)

$$\begin{array}{rcr} \$\ 44 & : & 8\ \textit{pizzas} \\ \downarrow\ \times 2.5 & & \times 2.5\ \downarrow \\ \boxed{\$\ 110} & : & 20\ \textit{pizzas} \end{array}$$

(d) Notice that he will grade just 18 papers in the second round. So we don't want to add the original info about 20 students, but subtract 2 students from that:

8 hours for 20 students means 0.8 hours for 2 students (divide by 10).

8 *hours*	:	20 *students*
− 0.8 *hours*		− 2 *students*
7.2 hours	*for*	18 *students*

Direct scaling: Remember that we always find the scale factor by dividing FINAL number by initial number, not necessarily bigger by smaller.

(Scale factor is $18 \div 20 = 0.9$.)

8 *hours*	:	20 *students*
↓ × 0.9		× 0.9 ↓
72. hours	:	18 *students*

(e) Additive scaling:

5 *cups flour*	:	2 *cups sugar*
5 *cups flour*	:	2 *cups sugar*
5 *cups flour*	:	2 *cups sugar*
+ 2.5 *cups flour*	:	+ 1 *cup sugar*
17.5 cups flour	*for*	7 *cups sugar*

Direct scaling: (Scale factor is $7 \div 2 = 3.5$.)

5 *cups flour*	:	2 *cups sugar*
↓ × 3.5		× 3.5 ↓
17.5 cups flour	*for*	7 *cups sugar*

(f) Additive scaling:

2/3 *cup sugar*	:	6 *dozen candies*
2/3 *cup sugar*	:	6 *dozen candies*
+ 2/3 *cup sugar*	:	+ 6 *dozen candies*
2 *cups sugar*	*makes*	18 *dozen candies*
+ 2 *cups sugar*	:	+ 18 *dozen candies*
4 *cups sugar*	*makes*	36 dozen candies

Direct scaling: (Scale factor is $4 \div \frac{2}{3} = 6$.)

2/3 *cup sugar*	:	6 *dozen candies*
↓ × 6		× 6 ↓
4 *cups sugar*	*makes*	36 dozen candies

2. (a)

$\frac{1}{2}$ *dozen*	:	\$ 1.80
↓ × 2		× 2 ↓
1 *dozen*	:	\$ 3.60
↓ × 4.5		× 4.5 ↓
$4\frac{1}{2}$ *dozen*	*cost*	\$ 16.20

(b)

$$\begin{array}{ccc} 1\frac{1}{2}\ \textit{inches} & : & 12\ \textit{miles} \\ \downarrow \ \div 1\frac{1}{2} & & \div 1\frac{1}{2}\ \downarrow \\ 1\ \textit{inch} & : & \$\ 8\ \textit{miles} \\ \downarrow \ \times 6 & & \times 6\ \downarrow \\ 6\ \textit{inches} & \textit{is} & \boxed{48\ \text{miles}} \end{array}$$

(c)

$$\begin{array}{ccc} \$\ 44 & : & 8\ \textit{pizzas} \\ \downarrow \ \div 8 & & \div 8\ \downarrow \\ \$\ 5.50 & : & 1\ \textit{pizza} \\ \downarrow \ \times 20 & & \times 20\ \downarrow \\ \boxed{\$\ 110} & \textit{for} & 20\ \textit{pizzas} \end{array}$$

(d)

$$\begin{array}{ccc} 8\ \textit{hours} & : & 20\ \textit{students} \\ \downarrow \ \div 20 & & \div 20\ \downarrow \\ 0.4\ \textit{hours} & : & 1\ \textit{student} \\ \downarrow \ \times 18 & & \times 18\ \downarrow \\ \boxed{7.2\ \text{hours}} & \textit{for} & 18\ \textit{students} \end{array}$$

(e)

$$\begin{array}{ccc} 5\ \textit{cups flour} & : & 2\ \textit{cups sugar} \\ \downarrow \ \div 2 & & \div 2\ \downarrow \\ 2.5\ \textit{cups flour} & : & 1\ \textit{cup sugar} \\ \downarrow \ \times 7 & & \times 7\ \downarrow \\ \boxed{17.5\ \text{cups flour}} & \textit{for} & 7\ \textit{cups sugar} \end{array}$$

(f)

$$\begin{array}{ccc} 2/3\ \textit{cup sugar} & : & 6\ \textit{dozen candies} \\ \downarrow \ \div 2/3 & & \div 2/3\ \downarrow \\ 1\ \textit{cup sugar} & : & 9\ \textit{dozen candies} \\ \downarrow \ \times 4 & & \times 4\ \downarrow \\ 4\ \textit{cups sugar} & \textit{for} & \boxed{36\ \text{dozen candies}} \end{array}$$