Math 210 - Dr. Miller - Homework #2: Mixed Problem-Solving Strategies

1. Solve each problem below, explaining *briefly* but clearly and in mathematically meaningful language.
   - (a) Peter and Jill each worked a different number of days but earned the same amount of money. Determine how many days each person worked if Peter earns \$20 per day, Jill earns \$30 per day, and Peter worked 5 days more than Jill did.
   - (b) List the 3-digit numbers that can be written using each of the digits 2,5,8 exactly once.
   - (c) The product of 2 whole numbers is 96 and their sum is less than 30. What are the possibilities for the two numbers?
   - (d) Ellis went to the used book sale. Paperbacks were priced at 10 cents each and hardbacks were 50 cents. He got 18 books and paid \$3.40. How many were hardbacks?
   - (e) A ball is dropped from a height of 176 ft. It bounces upward 88 ft., and each time it returns upward, it goes half as high as on the previous bounce. How far is the ball off the ground after it has bounced a total of 400 ft?
   - (f) Vic is half as old as I am. Mike is twice as old as I. The sum of all our ages is 49. How old is Mike?
   - (g) Danila's a biologist catching and banding birds. She has small metal bands in three colors: red, blue, yellow. She plans to put two bands on each bird's right leg, one above the other; reusing colors is fine. How many different band patterns are available?
   - (h) Mandy has a weekend business repairing toy vehicles. Right now she has a lot of bikes and trikes in her workshop to repair for charity. They have 27 seats and 60 wheels altogether. How many of each does she have?
2. Now solve these additional problems, again giving brief explanations: (My solutions are much more in-depth than yours are required to be, for mine show how Polya's Four Steps are actually used with-in the problem.)
   - (a) George has made up a sort of game to practice his addition facts. He notices that, if he wants to use all different digits each time, he can write the number 6 as $1 + 5$, $2 + 4$, or $1 + 2 + 3$. Since he's very comfortable with the commutative property his teacher taught him, he doesn't want to count the same numbers when they're only in a different order. In how many ways can he express 7 as a sum in his game? What about 10?
   - (b) Refer to Mandy's workshop in Problem #1h above. Usually, her workshop isn't so full, and she repairs wagons as well as bikes and trikes. Sometimes she has some of each to work on; other times, like with the charity work, she is lacking some vehicles. Right now, she has 5 vehicles in her workshop, and they have a total of 14 wheels. How many of each kind of toy vehicle might she have? (Find all possible answers.)

1. (b) Begin listing numbers that start with the 2: 258 and 285. Continue with those starting with the 5: 528, 582. Finally list the ones starting with the 8: 825, 852. We organized all options for each possible starting digit, so those six three-digit numbers are the complete list of answers.

   (c) Find all pairs of numbers whose product is 96 and also record their sum, using a table:

Number Pair	*Sum*
1 *and* 96	97
2 *and* 48	50
3 *and* 32	35
4 *and* 24	28
6 *and* 16	22
8 *and* 12	20
12 *and* 8	*Oops*! *We've already seen this pair.*

   We counted systematically up beginning with a pair using the number 1, then 2, etc., so when we reach the repeated pair 12 and 8, we know there will be no more new pairs at all. Now check the sums: The pairs whose sum is less than 30 are 4 and 24, then 6 and 16, and finally 8 and 12.

   (d) Let's guess and check: If he bought half of each kind, that's $9 \cdot \$0.10 \ + \ 9 \cdot \$0.50 \ = \ \$5.40$ spent - way too much. He must have a lot more paperbacks. Try 12 paperbacks and only 6 hardbacks, for $12 \cdot \$0.10 \ + \ 6 \cdot \$0.50 \ = \ \$4.20$ – still too much. Try 15 paperbacks, 3 hardbacks, for $15 \cdot \$0.10 \ + \ 3 \cdot \$0.50 \ = \ \$3.00$ – too low now, but not by much. Try 14 paperbacks and 4 hardbacks, for $14 \cdot \$0.10 \ + \ 4 \cdot \$0.50 \ = \ \$3.40$. That's it. Answer: He had 4 hardbacks.

   (f) If you like algebra, go for it: $x \ = \ my\ age$, $(1/2)x \ = \ Vic's$, $2x \ = \ Mike's$. So our total ages create the equation $x + 0.5x + 2x = 49$, or $3.5x = 49$, and $x = 14$. That's my age; Mike is twice as old, so he's 28.

   (g) I'll list her options as I did the numbers in Problem #1b: First put red on top - that's red over red, red over blue, and red over yellow. Now blue on top: blue over blue, blue over red, blue over yellow. Finally yellow on top: yellow over yellow, yellow over red, yellow over blue. She has these 9 possibilties.

   (h) 6 trikes, 21 bikes (This is like the cow/chicken problem. Imitate that explanation.)

2. (a) Understand the Problem: We must realize that combinations such as $3 + 1 + 1 + 1$ that repeat a number shouldn't be used. Also, we shouldn't count $1 + 5$ as one combination and then $5 + 1$ as another. Finally, we should be aware that we're not limited to adding just 2 numbers each time.

   Devise a Plan: This seems perfect for listing or making a chart because we want to stay organized in order not to overlook or double-count any answers. (Afterwards, I notice that I also broke the problem into simpler ones or cases, to shrink the amount of work/options.)

   Carry It Out: Start by trying to make 7 by adding only two different numbers:

$$1 + 6, \quad 2 + 5, \quad 3 + 4$$

   are the only options. Now try to break each of those up into expressions using three different numbers by systemically breaking apart just one of the numbers at a time. For instance, $1 + 6$ can't have its 1 broken apart, but its 6 can be:

$$1 + 1 + 5, \quad 1 + 2 + 4, \quad 1 + 3 + 3$$

   but of these, the only one that counts is $1 + 2 + 4$. Trying this strategy on the other 2-number sums – $2 + 5$ and $3 + 4$ – doesn't produce any new expressions. Neither does trying to break down any of the 3-number answers into 4-number ones.

When you follow the strategy of starting with 2-number sums, then breaking just one of their numbers apart at a time to make 3-number sums, and finally looking for 4-number sums, you get these results for 10:

$$1 + 9, \quad 2 + 8, \quad 3 + 7, \quad 4 + 6$$

$$1 + 2 + 7, \quad 1 + 3 + 6, \quad 1 + 4 + 5, \quad 2 + 3 + 5$$

$$1 + 2 + 3 + 4$$

(When reasoning is identical to an earlier explanation in the same problem, as here for the case of 10 vs. 7, I don't require you to explain it in great detail yet again.)

Look Back: We only have to tell how many combinations we found: We found 4 different results that work for a sum of 7 and 9 different results creating a sum of 10.

(b) Understand the Problem: We need to know how many wheels each type of toy has. We also accept that it's okay for some toys not to be there at all, as in zero bikes this week.

Devise a Plan: I like the idea of a list again, to stay organized, and of breaking into simpler problems to cut down on the work.

Carry it out: There could be as many as 3 wagons without exceeding 14 wheels. Three wagons could only go with one bike, though, and that's not enough vehicles (only 4). So 3 wagons isn't a part of a correct answer.

Two wagons have 8 wheels altogether, leaving 6 wheels unaccounted for. That could be 2 trikes, but again, there wouldn't be enough vehicles altogether (only 4), so it must be 3 bikes giving us those 6 necessary wheels.

One wagon leaves 10 wheels unaccounted for, and only 4 vehicles to do it. If they were all bikes, that would not make the 10 wheels (only 8). Three bikes and a trike can't either (9 wheels). Two bikes and two trikes works. One bike and 3 trikes doesn't (11 wheels), nor does 4 trikes (12 wheels).

Finally, if there are no wagons, all 14 wheels belong to a total of 5 bikes and trikes. Five bikes can't do it (only 10 wheels), nor can 4 bikes and trike (11 wheels). Three bikes and 2 trikes is only 12 wheels. Two bikes and 3 trikes fails also (13 wheels). One bike and 4 trikes works, but all trikes will be too many wheels (15 wheels).

Look Back: We're asked for all possible answers, so here they are: One answer: *2 wagons, 3 bikes.* Another answer: *1 wagon, 2 bikes, 2 trikes.* Third answer: *1 bike, 4 trikes.*