1. Solve each problem below, explaining *briefly* but clearly and in mathematically meaningful language.
   (a) Ezra has some pennies, nickels, and dimes. He has half as many dimes as he does pennies and 3/4 as many nickels as dimes. His coins are worth \$1.26 altogether. How many of each coin does he have?
   (b) How many different combinations of nickels, dimes, and/or quarters can you make that are worth 55 cents?
   (c) Dan baked some cookies. Sam took half the cookies. Then Sue took half of the remaining cookies. Later, Lisa took half of the cookies that were left. When Dan came home, he saw only 3 cookies. How many cookies did Dan bake altogether? (Washington State Grade 4 Assessment)
   (d) Resolve Problem #1c above, but assume that Sam took two fifths of the cookies.
   (e) Repeat Problem #1c once more, but now assuming that Sue took five sevenths of the cookies.
   (f) A pier in the harbor has a ladder fastened to it. At high tide, there are 3 rungs showing. The rungs are 8 inches apart. At low tide, the water level sinks 20 feet. How many rungs of the ladder are showing now?
   (g) Consider this mathematical machine:

   $$\textit{Input} \longrightarrow \textit{Add } 7. \longrightarrow \textit{Multiply by } 6. \longrightarrow \textit{Subtract } 18. \longrightarrow \textit{Divide by } 2. \longrightarrow \textit{Output}$$

   What number would you have to use as input to get 39 as the output? To get 57 as the output?
   (h) At Tommy's Yogurt Shop, you can order either a plain vanilla yogurt or you can order it with one or more of 3 toppings: nuts, chocolate chips, and/or coconut. How many different combinations of yogurt dishes can Tommy's sell?

2. Now solve these additional problems, again giving brief explanations: (My solutions are much more in-depth than yours are required to be, for mine show how Polya's Four Steps are actually used with-in the problem.)
   (a) Find all two-digit numbers that equal three times the product of their digits.
   (b) Jeffery has some dimes and quarters. He spent 50 cents on Pepsi, half of his quarters on video games, and 35 cents for a Pokemon gadget. If he ended up with 85 cents, what combinations could he have started with? (Find all possible answers.)

1. (b) The correct answer is just "11." (You were not asked to list the combinations, but here they are: 2 quarters and a nickel, 1 quarter and 3 dimes, 1 quarter and 2 dimes and 2 nickels, 1 quarter and 1 dime and 4 nickels, 1 quarter and 6 nickels, 5 dimes and a nickel, 4 dimes and 3 nickels, 3 dimes and 5 nickels, 2 dimes and 7 nickels, 1 dime and 9 nickels, 11 nickels. Reason this out like the Mandy's workshop problem with the wagons.)

   (c) The answer is 24.

$$(24) \xrightarrow{\times\frac{1}{2}} (12) \xrightarrow{\times\frac{1}{2}} (6) \xrightarrow{\times\frac{1}{2}} 3$$

   (d) The answer is 20.

$$(20) \xrightarrow{\times\frac{3}{5}} (12) \xrightarrow{\times\frac{1}{2}} (6) \xrightarrow{\times\frac{1}{2}} 3$$

   We get to the point where 12 is *three* fifths of the cookies, so create a unit fraction by dividing by 3 to get that a single fifth would be 4. That means that the total amount, all 5 fifths, is equal to $5 \cdot 4 = 20$.

   (f) Each 8-inch drop shows an additional rung. 20 feet equals 240 inches, so that's 30 groups of 8 inches. Answer: 33 rungs are showing.

   (g) 9 - Compute $39 \cdot 2$, then restore the 18 (to get 96). Divide by the 6 and finally subtract the 7. 15 – repeat the steps: $57 \times 2 = 114$, plus 18 is 132, divided by 6 is 22, minus 7 is 15.

   (h) Total up

$$1\ (plain)\ +\ 3\ (with\ one\ topping)\ +\ 3\ (with\ two\ toppings)\ +\ 1\ (everything)\ =\ 8$$

2. (a) Understand the Problem: You have to know what a product is. You are also restricted to just 2-digit numbers.

   Devise a Plan: I think I can just list all (90!) 2-digit numbers and check them, but maybe I'll find a pattern to shorten the work.

   Carry It Out: I can tell that there's no need to check the numbers 10, 20, and so on that end in zero, because the product of their digits will be 0, and there's no way any of those numbers are equal to 3×0. So I've ruled all of those out. Now I'll make a list of what's left, the 2-digit number on the left, 3 times both digits on the right, and a question mark between until I check whether they're equal.

$$\begin{array}{lcll} 11 & \stackrel{?}{=} & 3 \cdot 1 \cdot 1 & \quad no \\ 12 & \stackrel{?}{=} & 3 \cdot 1 \cdot 2 & \quad no \\ 13 & \stackrel{?}{=} & 3 \cdot 1 \cdot 3 & \quad no \end{array}$$

   This will take forever! Is there a shortcut? Well, if my number will be 3 times something, I only need to check 2-digit numbers that are already multiples of 3:

$$\begin{array}{lcll} 15 & \stackrel{?}{=} & 3 \cdot 1 \cdot 5 & \quad yes! \\ 18 & \stackrel{?}{=} & 3 \cdot 1 \cdot 8 & \quad no \\ 21 & \stackrel{?}{=} & 3 \cdot 2 \cdot 1 & \quad no \\ 24 & \stackrel{?}{=} & 3 \cdot 2 \cdot 4 & \quad yes! \\ 27 & \stackrel{?}{=} & 3 \cdot 2 \cdot 7 & \quad no \\ 30 & & oh,\ right - skip\ it & \\ 33 & \stackrel{?}{=} & 3 \cdot 3 \cdot 3 & \quad no \\ 36 & \stackrel{?}{=} & 3 \cdot 3 \cdot 6 & \quad no \end{array}$$

39	$\stackrel{?}{=}$	$3 \cdot 3 \cdot 9$	*no*
42	$\stackrel{?}{=}$	$3 \cdot 4 \cdot 2$	*no*
45	$\stackrel{?}{=}$	$3 \cdot 4 \cdot 5$	*no*
48	$\stackrel{?}{=}$	$3 \cdot 4 \cdot 8$	*no*
51	$\stackrel{?}{=}$	$3 \cdot 5 \cdot 1$	*no*
54	$\stackrel{?}{=}$	$3 \cdot 5 \cdot 4$	*no*
57	$\stackrel{?}{=}$	$3 \cdot 5 \cdot 7$	*no*
63	$\stackrel{?}{=}$	$3 \cdot 6 \cdot 3$	*no*
66	$\stackrel{?}{=}$	$3 \cdot 6 \cdot 6$	*no*
69	$\stackrel{?}{=}$	$3 \cdot 6 \cdot 9$	*no*
72	$\stackrel{?}{=}$	$3 \cdot 7 \cdot 2$	*no*
75	$\stackrel{?}{=}$	$3 \cdot 7 \cdot 5$	*no*
78	$\stackrel{?}{=}$	$3 \cdot 7 \cdot 8$	*no*
81	$\stackrel{?}{=}$	$3 \cdot 8 \cdot 1$	*no*
84	$\stackrel{?}{=}$	$3 \cdot 8 \cdot 4$	*no*
87	$\stackrel{?}{=}$	$3 \cdot 8 \cdot 7$	*no*
93	$\stackrel{?}{=}$	$3 \cdot 9 \cdot 3$	*no*
96	$\stackrel{?}{=}$	$3 \cdot 9 \cdot 6$	*no*
99	$\stackrel{?}{=}$	$3 \cdot 9 \cdot 9$	*no*

Look Back: My method was systematic enough not to miss any, so I believe I've found them all. The answers are 15 and 24.

(b) Understand the Problem: It sounds like just a "work backwards" problem, but it's asking for all possible answers, so we can't quit once we get just one. It also wants to know about dimes and quarters separately, not total money amounts.

Devise a Plan: I'll use a typical "work backwards" diagram and try to check that I've considered all options.

Carry It Out: When you work backwards here, separate each box into quarters and dimes - sort of a mini-table - to keep the options straight. For instance, the 85 cents at the end might have been 3 quarters and a dime, or it could have been 1 quarter and 6 dimes. Each option creates its own work backwards diagram for us to trace through. (I can't typeset these.)
For instance, if he ended with 3 quarters, 1 dime, let's restore the 35 cents he spent on Pokemon by restoring 1 quarter, 1 dime. That has him at 4 quarters, 2 dimes right now. Next, restore the half of the quarters that was spent by doubling the quarters. So now he's at 8 quarters, 2 dimes. Finally, restore the 50 cents from the Pepsi, either as 2 quarters, or as 5 dimes. The two answers this gives are 10 quarters, 2 dimes or 8 quarters, 7 dimes.
Let's check the other option, where he ended with 1 quarter and 6 dimes. We still restore the Pokemon money with 1 of each coin, putting him at 2 quarters, 7 dimes. Doubling the quarters now gives 4 quarters, 7 dimes, and when we restore the Pepsi money in the two different ways described above, we get either 6 quarters, 7 dimes or 4 quarters, 12 dimes.

Look Back: There are four answers: *10 quarters, 2 dimes* or *8 quarters, 7 dimes* or *6 quarters, 7 dimes* or *4 quarters, 12 dimes.*