Math 210 - Dr. Miller - Homework #36: Prime Numbers

1. Illustrate the Prime Number Test in determining whether each number below is prime.
   (a) 483
   (b) 691
   (c) 851
2. Find the prime factorization of each number below.
   (a) 282
   (b) 5460
   (c) 36,900
   (d) $6300 \cdot 4500$
3. Find the prime factorization of each number below, writing your answer in simplest exponential form.
   (a) $2^{30} \cdot 3^2 \cdot 7 \cdot 9$
   (b) 36^{10}
   (c) $25^{20} \cdot 10^4$
4. How many divisors does each number below have? List all those that are less than 50.
   (a) 2^{10}
   (b) $2^{10} \cdot 7^{50}$
   (c) 5460
   (d) 36,900
   (e) $2^2 \cdot 15^6$
5. (a) In a 1-500 Sieve of Eratosthenes, what is the largest number that causes any crossing out?
   (b) What is the largest number that your answer in part (a) causes to be crossed out?
   (c) In a 1-800 Sieve, what is the largest number that causes any crossing out?
   (d) What is the largest number that your answer in part (c) causes to be crossed out?
   (e) In a 1-3,000 Sieve, what is the largest number that causes any crossing out?

1. (a) The square root of 483 is about 22.0. The primes less than or equal to this number are 2, 3, 5, 7, 11, 13, 17, 19.

$$\begin{array}{rcl} 2 & \nmid & 483 \\ 3 & \mid & 483 \end{array}$$

Stop checking: 483 is composite.

(b) The square root of 691 is about 26.3. The primes less than or equal to this number are 2, 3, 5, 7, 11, 13, 17, 19, 23.

$$\begin{array}{rcl} 2 & \nmid & 691 \\ 3 & \nmid & 691 \\ 5 & \nmid & 691 \\ 7 & \nmid & 691 \\ 11 & \nmid & 691 \\ 13 & \nmid & 691 \\ 17 & \nmid & 691 \\ 19 & \nmid & 691 \\ 23 & \nmid & 691 \end{array}$$

We've checked all options and they fail, so 691 is prime.

(c) The square root of 851 is about 29.1. The primes less than or equal to this number are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

$$\begin{array}{rcl} 2 & \nmid & 851 \\ 3 & \nmid & 851 \\ 5 & \nmid & 851 \\ 7 & \nmid & 851 \\ 11 & \nmid & 851 \\ 13 & \nmid & 851 \\ 17 & \nmid & 851 \\ 19 & \nmid & 851 \\ 23 & \mid & 851 \end{array}$$

Stop checking: 851 is composite.

2. (a) $282 = 2 \cdot 3 \cdot 47$

(b) $5460 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \cdot 13$

(c) $36900 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 41$

(d) $6300 \cdot 4500 = (2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7) \cdot (2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5) = 2^4 \cdot 3^4 \cdot 5^5 \cdot 7$

3. (a) $2^{30} \cdot 3^2 \cdot 7 \cdot 3^2 = 2^{30} \cdot 3^4 \cdot 7$
   (b) $(2^2 \cdot 3^2)^{10} = 2^{20} \cdot 3^{20}$
   (c) $(5^2)^{20} \cdot (2 \cdot 5)^4 = 5^{40} \cdot 2^4 \cdot 5^4 = 2^4 \cdot 5^{44}$

4. (a) 2^{10} has $10 + 1 = 11$ divisors. The ones less than 50 are:

$$1, \quad 2^1, \quad 2^2, \quad 2^3, \quad 2^4, \quad 2^5,$$

   or 1, 2, 4, 8, 16, and 32.

   (b) $2^{10} \cdot 7^{50}$ has $(10+1)(50+1) = 11 \cdot 51 = 561$ divisors. The ones less than 50 are:

$$1, \quad 2^1, \quad 2^2, \quad 2^3, \quad 2^4, \quad 2^5, \quad 7^1, \quad 2^1 \cdot 7^1, \quad 2^2 \cdot 7^1, \quad 7^2$$

   or 1, 2, 4, 7, 8, 14, 16, 28, 32, and 49.

   (c) $5460 = 2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 13$ has $(2+1)(1+1)(1+1)(1+1)(1+1) = 48$ divisors. The ones less than 50 are:

$$1, \quad 2^1, \quad 2^2, \quad 3^1, \quad 2^1 \cdot 3^1, \quad 2^2 \cdot 3^1,$$

$$5^1, \quad 2^1 \cdot 5^1, \quad 2^2 \cdot 5^1, \quad 3^1 \cdot 5^1, \quad 2^1 \cdot 3^1 \cdot 5^1,$$

$$7^1, \quad ,2^1 \cdot 7^1, \quad 2^2 \cdot 7^1, \quad 3^1 \cdot 7^1, \quad 2^1 \cdot 3^1 \cdot 7^1, \quad 5^1 \cdot 7^1,$$

$$13^1, \quad 2^1 \cdot 13^1, \quad 3^1 \cdot 13^1,$$

   or 1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 14, 15, 20, 21, 26, 28, 30, 35, 39, and 42.

   (d) $36900 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 41 = 2^2 \cdot 3^2 \cdot 5^2 \cdot 41$ has $(2+1)(2+1)(2+1)(1+1) = 54$ divisors. The ones less than 50 are:

$$1, \quad 2^1, \quad 2^2, \quad 3^1, \quad 2^1 \cdot 3^1, \quad 2^2 \cdot 3^1, \quad 3^2, \quad 2^1 \cdot 3^2, \quad 2^2 \cdot 3^2,$$

$$5^1, \quad 2^1 \cdot 5^1, \quad 2^2 \cdot 5^1, \quad 3^1 \cdot 5^1, \quad 2^1 \cdot 3^1 \cdot 5^1, \quad 3^2 \cdot 5^1, \quad 5^2 \quad 41,$$

   or 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 30, 36, 41, and 45.

   (e) $2^2 \cdot 15^6$ does NOT have 21 factors. You must prime factor first: $2^2 \cdot 3^6 \cdot 5^6$ has $3 \cdot 7 \cdot 7 = 147$ factors. The ones less than 50 are: 1, 2, 3, 4, 5, 6, 9, 10, 12, 18, 20, 25, 27, 30, 36, 45.

5. (a) $\sqrt{500} \approx 22.4$, so prime number 19 is the largest number to cause any crossing out.
   (b) We need the largest multiple of 19 and higher primes that's in this grid: $19 \times 19 = 361$, $19 \times 23 = 437$, and $19 \times 29 = 551$. The largest of these that's still in the 1-500 range is 437.
   (c) $\sqrt{800} \approx 28.3$, so prime number 23 is the largest number to cause any crossing out.
   (d) We need the largest multiple of 23 and higher primes that's in this grid: $23 \times 23 = 529$, $23 \times 29 = 667$, $23 \times 31 = 713$, and $23 \times 37 = 851$. The largest of these that's still in the 1-800 range is 713.
   (e) $\sqrt{3000} \approx 54.8$, so prime number 47 is the largest number to cause any crossing out.