1. **Scaling** is typically a mental technique, used when we think about how many times more (or less) one scenario is than another. We get that information by comparing the two GIVEN amounts that are the **same type** as each other.

   Example - On Chelsea's map, 1/4 inch equals 10 miles. She sees that Youngstown and East Liverpool are 3/4 of an inch apart. "That's 3 times a quarter-inch," she reasons, "so they are 3 times 10 miles – 30 miles – apart."

   Notice how she determines how many quarter inches there are in 3/4 of an inch. She's comparing inches, the GIVEN items that are the same type as each other (I say given because we know how many inches are being talked about in each of the comparisons in the problem: on the map AND between the cities. However the miles are NOT given in the actual city comparison.) She then applies the same comparison ("3 times") to the other type of item, the miles. Chelsea used *direct* scaling here, because she determined exactly how much to multiply the original 1/4 inch by to get to the target of 3/4 of an inch. The following is also scaling:

   Example - Dan used the same map, where 1/4 inch equals 10 miles. He too sees that Youngstown and East Liverpool are 3/4 of an inch apart. "That's 1/4, 10, 2/4, 20, 3/4, 30 : 30 miles apart," he counts.

   In Dan's case, he repeatedly added/counted the original ratio in order to reach his target. This is a repeated addition perspective, but because repeated addition IS multiplication, Dan's approach is definitely a form of scaling. One could also argue that he is using representative sets at the same time, because he just keeps adding on another copy of the original ratio each time.

2. **Unit-rate** reasoning is also a mental technique, applied when we start off by asking how much is needed for just **one** item. It's almost always followed by scaling. In a unit rate approach, we get our initial information by comparing the two items or amounts that are **different types** from each other, and we often divide (although sometimes multiplying is needed instead).

   Example - On Kevin's map 1/2 inch equals 10 miles. He sees that Columbus and Zanesville are 3-1/2 inches apart. "If half an inch is 10 miles," he decides, "then a whole inch is 20 miles. They're 3.5 whole inches apart, so that's 3.5 times 20 – 70 miles – apart in reality."

   (Even though Kevin applied scaling in his last sentence, his thinking is still an example of unit-rate reasoning because of his initial conversion to thinking about one inch.)

   Be careful: unit-rate reasoning doesn't require just the appearance of the number 1 somewhere along the way. Rather, we concentrate on getting 1 of the "**fully described**" items – that is, the type of units or amounts that are given **twice**. In Kevin's situation, inch numbers are given twice, so they're "fully described," and he paused to think about just one inch. Thinking about just one mile wouldn't have helped him at all.

   Example - Jayna used the same map, where 1/2 inch equals 10 miles. She too is looking at Columbus and Zanesville, which are 3-1/2 inches apart. "I wrote 10 miles over .5 inches first," she explains, "then doubled the top and bottom to get 20 miles per inch. But now I need to multiply both by 3.5, and that gives me 70 miles over 3.5 inches, so they're 70 miles apart."

   Jayna chose to write her ratios as fractions, and she put the units she wanted to work "per" – the units she wanted to think about ONE of – in the denominator. She observes that she multiplied by 2 to find how many miles per inch she had, but it's also true that she divided $10 \div 0.5$, and this will fit with children's experience that finding a unit rate as a fraction always means you divide the top number by the bottom.

3. Creating a **proportional equation** is a written process, completed using cross-multiplication.

   On Diane's map, the legend has been torn and simply says "1/2 inch equa...." She knows that Breezewood and Cranberry are about 130 miles apart in reality, and she sees that they're 2.6 inches apart on her map. She writes a proportional equation

   $$\frac{0.5\ in}{x} = \frac{2.6\ in}{130\ miles}$$

   and cross-multiplies to get $2.6x = 65$*, so* $x = 25$*. She writes her answer as 25 miles to include correct units of measurement.*

   Creating a carefully-labelled chart makes it easier to pair up the numbers correctly. The chart will have two rows and two columns. The columns can be labelled with the types of things being compared in the problem; in this case, miles and inches would work. The columns can be labelled by HOW the miles and inches are being compared: in this case, first it's information that comes from the legend, and then it's information that comes from the actual two cities.

	info from legend	*info from cities*
miles	x	130
inches	0.5	2.6

   Write a fraction bar to separate the two numbers in each column, and an equal sign between the columns and, voila!, proportional equation:

   $$\frac{x}{0.5} = \frac{130}{2.6}$$

   While different labelings for your charts may create equations that look different initially, they will always turn out to be identical once you cross-multiply.

4. Drawing **representative sets** gives a visual way to set up scaling or unit-rate, and sometimes even proportional equations.

   Cady reads "The ratio of children to adults on the bus is 5 to 2. There are 15 children on the bus." She draws one set of C, C, C, C, C, A, A and writes "C stands for 1 child, and A stands for 1 adult." Then she draws another C, C, C, C, C, A, A, and yet one more C, C, C, C, C, A, A. "Now I have 15 children on the bus," she announces.

   While children would literally draw rather than using letter abbreviations, they can repeat the representative sets - Cady's is the single C, C, C, C, C, A, A that she wrote first - until some desired target is reached. Here, it's the target of 15 children altogether, but this methods would work just as well in a situation where we were told how many adults were on the bus instead, or how many people in total.

   A representative sets diagram can also be used to answer a broader variety of questions than scaling or unit-rate, and even more than proportional equations sometimes. For instance, in Cady's problem, she can now answer questions such as "How many people are on the bus altogether?" or "How many more children are on the bus than adults?"