Cell phones must be turned off entirely - no ringers, no vibrating (even that is noticeable) - to provide a nondistracting environment for all.

1. [8 pts - 4 each] Name a different strategy that could be reasonably attempted for each problem below. Justify WHY that strategy applies, but do not solve nor describe how to solve.

   (a) When you drop a SuperBall ball, it always rebounds to half its previous height. If you drop it from the top of a 768-foot building, how high off the ground is it when it’s bounced for a total of 2,240 feet?

   (b) If you expanded the number \((333333333333334)^2\), how many 5’s would there be in the result?

2. [8 pts - 4 each] Find a formula for the \(n\)th term of each sequence below. Show sufficient clear work; if showing ample work is not possible, describe your reasoning verbally.

   (a) 95, 92, 89, 86, ...

   (b) 2, 8, 18, 32, ...

3. [5 pts] Bob gave \(3/8\) of his candy canes to Rachel, 12 to Jessica, and half of what was left to Micah, leaving him with 24. How many did he start with? Show clear work, but you need not explain.
4. Consider this Venn diagram:

(a) [2 pts] In which region(s) could you put cookies that are decorated with green icing but not colored sugar?

(b) [4 pts] There are 6 cookies that only have colored sugar, none in Region V, and twice as many in Region VI as in Region II. If there are 30 cookies with colored sugar altogether, how many of those don’t have green icing either? Either show work or explain your reasoning.

5. [4 pts] If possible, make up sets $P$ and $Q$ so that $n(P) = 3$ and $n(P \times Q) = 8$. If not possible, clearly explain why not.

6. Consider these sets for the entire problem:
   $U = \{1, 1_2, 2, 2_2, 3, \ldots, 7_2, 8\}$, $A = \{1, 3, 5, 7\}$, $B = \{x \mid x > 6\}$, $C = \{1, 1_2, 1, 1_2\}$

(a) [2 pts] Use the symbol $\in$ correctly in a symbolic statement about $C$.

(b) [4 pts] How many subsets does $B$ have? Clearly explain your claim.

(c) [6 pts] Find $n(\overline{A} \cap B)$. Show work.
7. [3 pts] Give the names of all terms involved in the number sentence $20 \div 2 = 10$. Spell correctly, and clearly identify which term goes with which number(s).

8. [2 pts] Use the constructivist definition to complete the following statement:

$$3 \times 4 = 12 \text{ because...}$$

9. [4 pts - 2 each] Make up a word problem clearly requiring each computation and model.

(a) grouping/repeated subtraction for $36 \div 4$

(b) missing addend for $15 - 8$

10. [4 pts] Identify the computation that cannot be performed, then thoroughly explain why not:

$$12 \div 0 \quad \text{or} \quad 0 \div 12$$

11. (a) [4 pts] Clearly explain what it means for a set to be closed under subtraction.

(b) [6 pts] Clearly explain what it means to say that an operation is associative; give a supporting example also.

12. [6 pts - 2 each] Identify which property or concept is being demonstrated in each number sentence below. Spell correctly.

(a) $5 \cdot (8 \cdot 2 + 4 \cdot 7) = 5 \cdot 8 \cdot 2 + 5 \cdot 4 \cdot 7$

(b) $5 \cdot (8 \cdot 2 + 4 \cdot 7) = 5 \cdot (4 \cdot 7 + 8 \cdot 2)$

(c) $(5 \cdot 8) \cdot 2 + 5 \cdot (4 \cdot 7) = (5 \cdot 8) \cdot 2 + (5 \cdot 4) \cdot 7$
13. \( [6 \text{ pts} - 3 \text{ each}] \) Write the three numbers that immediately follow the given one, in the same base.

   (a) \( 456_{\text{seven}} \)

   (b) \( TET_{\text{twelve}} \)

14. \( [4 \text{ pts}] \) Convert \( 12503_{\text{six}} \) into base ten; show clear work.

15. \( [5 \text{ pts}] \) Make up a 5-digit number that is a multiple of 11 and 12, but not 5. Demonstrate all appropriate divisibility tests on your number, but you need not explain.

16. Perform the indicated operation \textit{entirely} in the given base or system of numeration, showing all “borrows,” “carries,” or other necessary notations. You may use any algorithm you like in each case.

   (a) \( [3 \text{ pts}] \) MCDVI plus CMXCIX

   (b) \( [5 \text{ pts}] \) \( 154_{\text{six}} + 213_{\text{six}} + 345_{\text{six}} + 414_{\text{six}} + 503_{\text{six}} \)
17. (a) [3 pts] List all the factors of 210.

(b) [4 pts] Find the complete prime factorization of 23460; show all necessary work.

(c) [6 pts] Use any method to find the GCD and LCM of 23460 and 210. Show clear work, and indicate which is which.
18. [9 pts - 3 each] Identify each statement as true or false, carefully justifying your claim with the correct type of argument.

(a) $30 \mid 6$

(b) The LCM of two numbers is always larger than either of them.

(c) There is a number that has 15 as a factor, but not 5.

19. [6 pts] Warren raked yards and cleaned gutters all last month for $30$ a household. He spent all his well-earned rewards on some cool computer games he really wanted, at $35$ apiece. What’s the smallest number of CDs he could have bought? Show work and briefly explain.

20. (a) [3 pts] Write the exact number sentence modelled by this “mailtime” story: The mail carrier brings a bill for $46$ and 2 checks for $19$ each. He also takes away a misdelivered bill for $57$ and a check for $19$. What’s your net worth?

(b) [2 pts] Suppose in the above scenario that the mail carrier also takes away an illegible item. If your net worth is now $125$, what type of item did he remove, and for how much?

21. [6 pts - 3 each] Refer to the numbers $a$, $b$, $c$, and $d$ plotted on the number line below, not necessarily to scale!

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-1  0  1  d
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Fill in each blank with the correct choice of $<$ or $>$, then clearly explain your reasoning in a short sentence.

(a) $(b - a)c \underline{<} 0$

(b) $|a - d| \underline{<} |bc|$