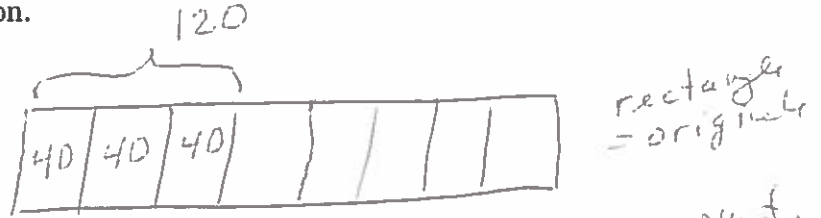


Work or explain in the space provided. This exam is worth 100 points.

- 1. [6 pts] Kendall handed out glow-sticks as allergy-safe treats for Halloween trick-or-treaters. At the end of the night, she had 120 glow-sticks left, which was $\frac{3}{8}$ of what she had started with. WITHOUT using algebra, determine how many she had at the start, and briefly explain the solution.

(Various) Diagram solution.



Since 120 was $\frac{3}{8}$ of her original amount, each eighth of the original is one of 3 fair shares in 120, or 40!

So all 8 eighths = $8 \times 40 = 320$ glow sticks

- 2. (a) [3 pts] Use correct notation to list $G \times H$ for sets $G = \{1, 3, 5\}$ and $H = \{\Delta\}$.

$$G \times H = \{(1, \Delta), (3, \Delta), (5, \Delta)\}$$

- (b) [4 pts] Make up your own representative sets to illustrate the set-based definition of $3+4$. Show correct notation, and explain in 1-2 sentences how the sets represent all the numbers involved in this situation, including the answer.

My set A represents 3 because it has 3 elements: $A = \{1, 2, 3\}$.

My set B represents 4 since B has 4 elements: $B = \{+, -, \times, \div\}$.

A and B are disjoint, and $A \cup B = \{1, 2, 3, +, -, \times, \div\}$ which has 7 elements.

The set-based definition says that under these conditions, $n(A \cup B) = n(A) + n(B)$ and yes, $7 = 3 + 4$.

① labeled pix + no else
 ② no expl.
 ③ algebra
 ④ no motivation for $120 \div 3$ vs. $120 \div 8$.
 ⑤ to ② skipped steps.
 ⑥ wrong answer.

⑦ no parens
 ⑧ no > between pairs
 ⑨ more list 1Δ, 3Δ, 5Δ.
 ⑩ no show $A \cup B$.
 ⑪ lost repr. idea.
 ⑫ no \cup notation.
 ⑬ $A \cup B$ not shown.
 ⑭ no expl.
 ⑮ mult.
 ⑯ pix w/ perfect discus

3. (a) [2 pts] Write a complete Fact Family that uses the numbers 2, 3, and your choice of an appropriate third number.

$2 + 3 = 5$ $5 - 3 = 2$
 $3 + 2 = 5$ $5 - 2 = 3$

OR

$2 \times 3 = 6$ $6 \div 3 = 2$
 $3 \times 2 = 6$ $6 \div 2 = 3$

OR

$1 + 2 = 3$ $3 - 2 = 1$
 $2 + 1 = 3$ $3 - 1 = 2$

(1) least one
(2) least two
on each

(b) [2 pts] Write the complete Fact Family that uses the numbers 7, 7, 0.

$7 + 0 = 7$ $7 - 7 = 0$
 $0 + 7 = 7$ $7 - 0 = 7$

4. [12 pts - 3 each] Make up a word problem to fit each computation and scenario below.

(a) $15 - 6$, comparison, using measurements (various)

Tony ran 15 kms today; Dennis ran only 6 kilometers (km). How much farther did Tony run?

(b) 4×8 , repeated addition, using objects (various)

There are 8 tables with 4 chairs each in our classroom. How many chairs are there altogether?

(c) $16 \div 8$, repeated subtraction, using objects (various)

Carrie baked 16 cupcakes + put them in containers of 8 each. How many containers did she need?

(d) $28 \div 4$, any scenario, using measurements (various)

Dig ran 28 miles over 4 days, the same number of miles per day. How many miles per day was that? (sharing/partitioning)

OR Dig recently ran 28 miles total over several days. If she ran 4 miles per day, how many days did it take? (repeated subtraction)

(1) miss add.
(1) obj
(2) take away
(3) neither
15-6 nor
15-9
(2) 15-9
(1) "final diff"
(3) not 4x8 mult.
(1) array
(1) 4 or 8 not showing.
(2) C. prod
(1) unclear.
(2) partition
(2) $16 \div 2$
(3) not \div

(2) obj
(3) not $28 \div 4$

5. [4 pts] Use the constructivist definition of division to clearly explain the answer/issue for "20 ÷ 0 =?"

- 3 no defn
- 1 no clear mult. shown.
- 2 20 ÷ 0 ≠ 0 discussion

If there's an answer to $20 \div 0$, it must check out:
 answer $\times 0 = 20$.
 yet no number can work as the unknown answer here to make this true.
 Thus, $20 \div 0$ is undefined/has no answer.

6. [6 pts - 2 each] Fill in each blank with the correctly spelled term.

- (a) In the number sentence $8+3 = 11$, the numbers 8 and 3 are called the addends.
- (b) In the number sentence $42 \div 6 = 7$, the number 42 is called the dividend, and the answer, 7, is called the quotient.

7. (a) [2 pts] Create a subtraction sentence in which the answer is the same as exactly one of the other two numbers. What is the correctly spelled term for the "answer" in your number sentence?

difference
 (varying sentence) $x - 0 = x$
 or $2x - x = x$.

(b) [2 pts] What is the correctly spelled term for the number that's NON-matching in your problem?

subtrahend or minuend, depending on sentence.

- 1 you can add digits. (w/o discuss what purpose)
- 2 can add #s
- 61

8. (a) [3 pts] What does it mean to say that a system of numeration is additive? Name one that is.

you simply add the values of all digits to find the overall value of the numeral.
 Egyptian is additive.

(b) [3 pts] Are using place value and being positional the same? Explain.

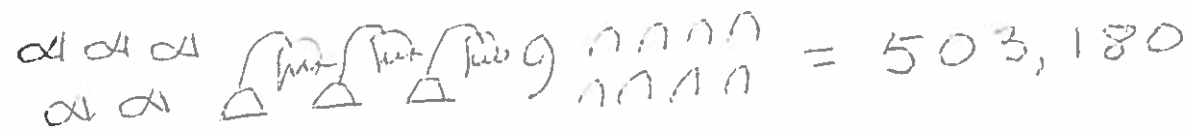
- 1 yes/not answered.
- 2 doesn't discuss ones at all.

No, not quite.
 The concept of place value can change the value of actual digits, changing the overall value of the numeral.
 Positional systems are influenced by position of digits but need not change value of digits themselves.

9. [3 pts] Counting by fifties, what are the next three Roman numerals after MCCCXLIV?

MCCCXCIV, MCDXLIV, MCDXCIV

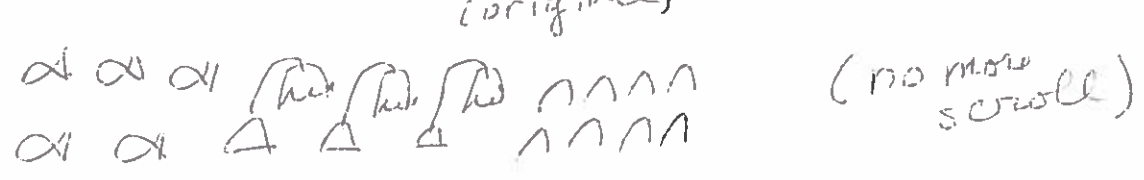
10. (a) [2 pts] What is the Egyptian numeral that uses 3 lotus flowers, 5 tadpoles, a scroll, and 8 heel bones worth in Hindu-Arabic?



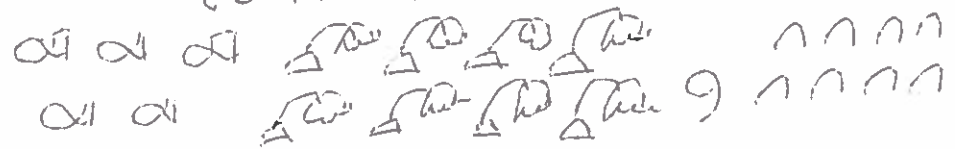
(b) [2 pts] What Egyptian numeral immediately precedes this one?



(c) [2 pts] What Egyptian numeral precedes it if we count by hundreds instead?



(d) [2 pts] What Egyptian numeral is 5,000 more than the original? (I.e., follows it if we count by 5,000s.)



11. (a) [2 pts] Convert to Hindu-Arabic: MMCMX

2,910

(b) [2 pts] What Roman numeral immediately follows this one?

MMCMXI

(c) [2 pts] What Roman numeral precedes the original if we count by hundreds?

MMDCXX

12. (a) [2 pts] Write the word form of 506,009.

Five hundred-six thousand, nine.

(b) [2 pts] Write its expanded form WITHOUT multiplication (2nd grade).

500,000 + 6,000 + 9

(-1) each bad value
(-1) not immedi.
(-2) 1st follows not Egyptian
(-2) each non-Egyptian
(-1) 2nd follows

(-1) 5 back not 5.

(-1) follows

(-1) and

(-1) multiple types
(-1) no +

13. [3 pts] Write the Roman numeral for 5,279.

VCCCLXXIX

Sub bar
w/arrives +
assignment
no concept
of bar

bad bar

each
error
incomplete
reason
not correct
reason

overall
no tell.

14. [6 pts - 2 each] Using ALL the digits in each list, make the largest and smallest legitimate Roman numerals, telling which is which. If not possible, say why.

(a) D,X,M,C,X Largest: MDCXX
Smallest: MCDXX

(b) I,I,I,X,I

impossible - the arrangement would have to be XIII, illegal.

(c) I,X,C,X,I

Largest: CXXII
Smallest: CXXII also.

15. [8 pts - 4 each] Convert the following numerals to base ten; show clear work.

(a) 506_{twenty-three}

various work
 $5 \cdot 23^2 + 0 \cdot 23 + 6$ base ten
 $= 2,645 + 0 + 6$ " "
 $= \boxed{2651_{ten}}$

(b) 21131_{five}

various work
 $2 \cdot 5^4 + 1 \cdot 5^3 + 1 \cdot 5^2 + 3 \cdot 5 + 1$
 $= 1250 + 125 + 25 + 15 + 1$
 $= \boxed{1,416_{ten}}$

16. [4 pts] Find the value of base b if $58_b = 303_{ten}$. Show work.

various
 $5b + 8 = 303$
 $5b = 295$
 $b = \boxed{59_{base ten}}$

Digits
of exp.
form.

Work or explain in the space provided. This exam is worth 100 points.

1. [6 pts] Kendall handed out $\frac{3}{8}$ of her Halloween glow-sticks to trick-or-treaters as part of the Teal Pumpkin Campaign. She counted that she had handed out 120 glow-sticks. WITHOUT using algebra, determine how many she had at the start, and briefly explain the solution.

See Version A.

2. (a) [3 pts] Use correct notation to list $G \times H$ for sets $G = \{1, 3, 5\}$ and $H = \{\Delta\}$.

See Version A.

- (b) [4 pts] Make up your own representative sets to illustrate the set-based definition of $3+4$. Show correct notation, and explain in 1-2 sentences how the sets represent all the numbers involved in this situation, including the answer.

See Version A.

Bp 2

16
16

3. (a) [2 pts] Finish the constructivist definition: $8 \times 9 = 72$ because ...

$8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = 72$

or $9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 = 72$

(-2) answer uses division

(b) [2 pts] Write the complete Fact Family that uses the numbers 7,7,1.

$7 \times 1 = 7$

$7 \div 7 = 1$

$1 \times 7 = 7$

$7 \div 1 = 7$

(-1) lost one
(-2) lost two or more

4. [12 pts - 3 each] Make up a word problem to fit each computation and scenario below.

(a) $15 - 6$, missing addend, using measurements (various)

Mary runs 15 miles every week. So far this week she has run 6 miles. How much farther will she run by week's end?

(-1) objects
(-2) take away
(-3) not 15-6. nor 15-9
(-2) 15-9

(b) 4×8 , Cartesian product, using objects

See Version A.

(-2) rep. add.
(-3) not mult.
(-2) array
(-2) not 4×8 .

(c) $16 \div 8$, repeated subtraction, using objects

See Version A.

(-2) partition
(-2) $16 \div 2$
(-3) not division.

(d) $28 \div 4$, any scenario, using measurements

See Version A.

(-3) not division
(-2) objects

5. [4 pts] Use the constructivist definition of division to clearly explain the answer/issue for "0 ÷ 20 =?"

-1 reversed logic: answer is 0, so it checks out.
-3 no defn
-2 use defn incorrectly! draws wrong concl. from it.
-2 target defn.

If there's an answer to $0 \div 20$, it must check out: answer $\times 20 = 0$. The number 0 works in the role/position of the answer.
So $0 \div 20 = 0$.

6. [6 pts - 2 each] Fill in each blank with the correctly spelled term.

-1 spell
-2 term

(a) In the number sentence $8 \times 3 = 24$, the numbers 8 and 3 are called the factors.
(b) In the number sentence $42 \div 6 = 7$, the number 6 is called the divisor, and the answer, 7, is called the quotient.

7. (a) [2 pts] In the number sentence $23 - 16 = 7$, what is the correctly spelled term for the "answer"?

-1 wrong #

(b) [2 pts] What is the correctly spelled term for either of the other numbers (and which did you pick)? 23 minuend, 16 subtrahend

8. (a) [3 pts] What does it mean to say that a system of numeration is positional? Name one that is.

-1 described place value.
-3 commutativity of number sentences
-1 no/bad name.

In a positional system, the locations of the digits — or rearranging the digits — can affect the overall value of the numeral.
Roman is. Hindu-Arabic is

(b) [3 pts] Are a number and a numeral the same thing? Explain.

-1 reversed numeral concept unclear
-1 number concept missing.
-2 number multi-digit numeral! bad concept.

Not quite. A number is just the idea of "how many," whereas a numeral refers specifically to how we write symbols to represent a number.

9. [3 pts] Counting by fives, what are the next three Roman numerals after DCCXXXIII?

-2 not by fives.

DCCXXXVIII, DCCXLIII, DCCXLVIII

10. (a) [2 pts] What is the Egyptian numeral that uses 3 lotus flowers, 5 tadpoles, a scroll, and 8 heel bones worth in Hindu-Arabic?

See Version A.

- (b) [2 pts] What Egyptian numeral immediately precedes this one?

Version A

- (c) [2 pts] What Egyptian numeral precedes it if we count by hundreds instead?

Version A

- (d) [2 pts] What Egyptian numeral is 5,000 more than the original? (I.e., follows it if we count by 5,000s.)

Version A.

11. (a) [2 pts] Convert to Hindu-Arabic: MCDX

1,410

- (b) [2 pts] What Roman numeral immediately follows this one?

MCDXI

- (c) [2 pts] What Roman numeral precedes the original if we count by hundreds?

MCCCX

12. (a) [2 pts] Write the word form of 506,009.

See Version A.

- (b) [2 pts] Write its expanded form WITHOUT multiplication (2nd grade).

See Version A.

8p5

20
20

(2) no bar

13. [3 pts] Write the Roman numeral for 10,504.

X̄DIV

See warm-ups on Topics list + assignment re: bar.

(1) bad bar.

14. [6 pts - 2 each] Using ALL the digits in each list, make the largest and smallest legitimate Roman numerals, telling which is which. If not possible, say why.

(a) D,X,M,C,X

(b) I,I,I,X,I

See Version A.

(c) I,X,C,X,I

15. [8 pts - 4 each] Convert the following numerals to base ten; show clear work.

(a) 506_{nineteen}

$$5 \cdot 19^2 + 0 \cdot 19 + 6 \cdot 1 = 1805 + 0 + 6 = 1811_{\text{ten}}$$

(b) 21131_{five}

See Version A.

16. [3 pts] Find the value of base b if $58_b = 303_{\text{ten}}$. Show work.

See Version A.

17. [10 pts] Find three Hindu-Arabic numerals that satisfy the entire list of clues below. Show supporting work, and clearly indicate your final answers.

- The numeral has exactly two 5s.
- The digits in the thousands period are all even, and their sum is 6.
- There are no digits in the 10^8 position or higher.
- There is only one 3, and it is in the leftmost position.
- The digit in the 10^1 position equals that in the ten thousands position.
- The sequence 320 appears in the numeral.

See
Version A.