Math 235 - Dr. Miller - HW #1: Logic Review - **Due Friday, 8-30-2024, by 4pm**

HW handouts in this course don't leave room for your work, so always work on your own paper, leaving room for my comments, and staple the question sheet to the front when finished.

1. What can be said - if anything - about the truth value of the statement form $(P \vee Q) \to (Q \leftrightarrow R)$ in each case below? Describe your reasoning in a sentence or two. (Do NOT make full truth tables.)
   (a) $P$ is true.
   (b) $Q$ and $R$ are both true.
   (c) $P$ and $Q$ are both false.
   (d) $P \leftrightarrow Q$ is true but $P \to R$ is false.

2. Again consider the statement form $(P \vee Q) \to (Q \leftrightarrow R)$.
   (a) If $P \leftrightarrow Q$ is false, is there a fixed truth value for $R$ that will make the above statement form true, or do we not have enough information? Explain your reasoning.
   (b) If $P \vee R$ is false, is there a fixed truth value for $Q$ that will make the above statement form false, or do we not have enough information? Explain your reasoning.

3. Negate the statements below; try to use simplest logical form (SLF) when meaningful.
   (a) $\sin A > 0$ and $\cos A = 0$.
   (b) Either of $2x + y = 0$ or $y \geq 3$ implies that $x \leq 1.5$.
   (c) If $x$ or $y$ is even, then $xy$ is even.
   (d) $\frac{b^2}{a} \in \mathbf{Z}$ if and only if $\frac{b}{a} \in \mathbf{Z}$.
   (e) $\sqrt{n} > n$ if $0 < n < 1$.
   (f) $\sin A$ and $\cos A$ are both positive only if $A$ is in Quadrant I.
   (g) There exists a function $f$ for which, if $x > 0$, then $f(x) < 0$ or $f(x) \geq 1$.
   (h) (Careful!) The equation $x^2 - ax + 3 = 0$ has a solution for some integer $a$.
   (i) There are real numbers $x$ and $y$ satisfying $x < y$ and $x^2 + y^2 = 1$.
   (j) For each $x \in \mathbf{R}$, there is $y \in R$ for which $xy > 1$.
   (k) There is a number $\delta > 0$ for which, for every $\epsilon > 0$, $\epsilon + \delta < 0$.
   (l) For all positive integers $p$, at most one of $p+2$, $p+4$, or $p+6$ is prime. (Negate "at most" meaningfully.)
   (m) There exists a set $S$ containing at least 10 consecutive composite numbers.

4. For each conditional statement below, identify its hypothesis, written as a stand-alone sentence (that is, with no conditional words remaining: for example, "Silver is a cat," not "if Silver is a cat.")
   (a) If $a$ and $b$ have different signs, then $ab < 0$.
   (b) $ab$ being positive implies that $|a + b| = |a| + |b|$.
   (c) $a$ can only be a multiple of $b^2$ if $a$ is a multiple of $b$.
   (d) It is sufficient that $c$ be negative for $a^4b^2c$ to be negative.
   (e) $a^3$ is positive if $a$ is positive.
   (f) $|a + b| = |a| + |b|$ only if $a$ and $b$ have the same signs.

5. Perform the 2-part tasks below for the indicated conditional variation, using the required synonyms. (Hint: it helps to identify the original hypothesis and conclusion clearly FIRST, which is what you did in Problem #4.)
   (a) Write the hypothesis of the converse of #4a, then write the full converse in if-then form.
   (b) Write the hypothesis of the inverse of #4e, then write the full inverse in if-then form.
   (c) Write the hypothesis of the contrapositive of #4b, then write the full contrapositive in if-then form.
   (d) Write the hypothesis of the converse of #4c, then write the full converse using the word "necessary."
   (e) Write the hypothesis of the inverse of #4f, then then write the full inverse using a trailing if.
   (f) Write the hypothesis of the contrapositive of #4d,then write the full contrapositive using "only if."