Math 235 - Dr. Miller - HW #1: Logic Review - **Due Monday, Jan. 22, by 4pm**

HW handouts in this course don't leave room for your work, so always work on your own paper, leaving room for my comments, and staple the question sheet to the front when finished.

1. What can be said - if anything - about the truth value of the statement form $(P \vee Q) \to (P \leftrightarrow R)$ in each case below? Describe your reasoning in a sentence or two. (Do NOT make full truth tables.)
   (a) P is true.
   (b) Q and R are both true.
   (c) P and R are both false.
   (d) $P \leftrightarrow Q$ is true but $P \to R$ is false.

2. Again consider the statement form $(P \vee Q) \to (P \leftrightarrow R)$.
   (a) If $P \leftrightarrow Q$ is false, is there a fixed truth value for R that will make the above statement form true, or do we not have enough information? Explain your reasoning.
   (b) If $P \vee R$ is false, is there a fixed truth value for Q that will make the above statement form false, or do we not have enough information? Explain your reasoning.

3. Negate the statements below; try to use simplest logical form (SLF) when meaningful.
   (a) $\sin A \geq 0$ and $\cos A = 0$.
   (b) $2x + y = 0$ or $y \geq 3$ implies that $x \leq 1.5$.
   (c) If x or y is even, then xy is even.
   (d) $\frac{b^2}{a} \in \mathbf{Z}$ if and only if $\frac{b}{a} \in \mathbf{Z}$.
   (e) $\sqrt{n} > n$ if $0 < n < 1$.
   (f) $\sin A$ and $\cos A$ are both positive only if A is in Quadrant I.
   (g) There exists a function f for which, if $x > 0$, then $f(x) < 0$ or $f(x) > 1$.
   (h) There are real numbers x and y satisfying $x < y$ and $x^2 + y^2 = 1$.
   (i) For each $x \in \mathbf{R}$, there is $y \in R$ for which $xy > 1$.
   (j) For every $\epsilon > 0$, there is a number $\delta > 0$ for which $\epsilon + \delta < 0$.
   (k) For all positive integers p, at most one of $p+2$, $p+4$, or $p+6$ is prime. (Negate "at most" meaningfully.)
   (l) There exists a set S containing at least 10 consecutive composite numbers.

4. For each conditional statement below, identify its hypothesis, written as a stand-alone sentence (that is, with no conditional words remaining: for example, "Silver is a cat," not "if Silver is a cat.")
   (a) If a and b have different signs, then $ab < 0$.
   (b) ab being positive implies that $|a + b| = |a| + |b|$.
   (c) a can only be a multiple of b^2 if a is a multiple of b.
   (d) It is necessary that c be negative for a^4b^2c to be negative.
   (e) a^3 is positive if a is positive.
   (f) $|a + b| = |a| + |b|$ only if a and b have the same signs.

5. Write the indicated conditional variation, using the required synonyms. (Hint: it helps to identify the original hypothesis and conclusion clearly FIRST, which is what you did in Problem #4.)
   (a) Write the converse of #4a in if-then form.
   (b) Write the inverse of #4e in if-then form.
   (c) Write the contrapositive of #4b in if-then form.
   (d) Write the converse of #4c using the word "sufficient."
   (e) Write the inverse of #4d using "only if."
   (f) Write the contrapositive of #4f using a trailing if.