Math 235 - Dr. Miller - HW #2, **PART A**: Direct Proof, Proof by Cases - Due by 4pm Friday, 9-6-2024

There will be additional problems assigned next week, but they will NOT be due until the following Monday.

Work on your own paper, leaving plenty of room for my comments, and staple this page to the front.

Correct proof structure always includes (1) assumptions, (2) NTS, (3) body, (4) conclusion, and (5) "exit move." Embed algebra/computations within sentences, not just as lists of equations like you might show on a calc or pre-calc problem. Finally, use FORMAL definitions when possible; childhood understanding of patterns is NOT acceptable reasoning in a proof.

1. Write complete, rigorous proofs of these statements:
   (a) Let $m, n \in \mathbf{Z}$. If $m$ is even and $n$ is odd, then $m - n$ is odd.

   (b) Let $x, y, z \in \mathbf{Z}$. If all of them are odd, then $xy + z^2$ is even.

   (c) Let $a, b, c \in \mathbf{Z}$. If $ab \mid c$, then $a \mid c$.

   (d) Let $x, y \in \mathbf{Z}$. If $15 \mid x$ and $10 \mid y$, then $5 \mid x^2 + 3y$.

   (e) Let $p, q \in \mathbf{R}$. If $p < q$, then $2p < 2q + 1$.

   (f) If $x, y \in \mathbf{Q}$, then $3x - (y/2) \in \mathbf{Q}$ also.