Math 235 - Dr. Miller - HW#2, Spring 2025 - **Due by 4pm Friday, 1-31-2025**

1. Rigorously prove the following statements directly.

   (a) Let $x, y, z \in \mathbf{Z}$. If all of them are odd, then $2y^2 - 5xz$ is odd.

   (b) Let $a, b, c \in \mathbf{Z}$. If $ab \mid c$, then $a \mid c^2$.

   (c) Let $x, y \in \mathbf{Z}$. If $12 \mid x$ and $20 \mid y$, then $4 \mid 3x^2 + y$.

2. **Proof by Cases** is based on this logical equivalence: If a disjunction among hypotheses can lead to a particular conclusion, then it must be that each hypothesis alone is capable of leading to that conclusion. That is,

$$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r).$$

   (a) Create a standard order truth table to confirm this equivalence, and point out in a sentence what behavior in the table actually shows that these statement forms are equivalent.

   (b) USE the equivalence to rewrite this statement about integers "If x is even or y is even, then xy is even" in equivalent form. (Your answer here should be a verbal statement representing the right-hand side of the equivalence.)

   (c) USE the equivalence to rewrite this statement about integers "If n has a remainder of 1 or 3 on division by 4, then n^2 has a remainder of 1."

3. Rigorously prove the following statements (directly).

   (a) If $c, d \in \mathbf{Z}$ have the same parity, then $c + d$ is even.

   (b) If $x, y \in \mathbf{R}$ with $|x + 3| = 1$ and $|y - 2| = 5$, then $x + y < 10$.

   (c) Let $x, y \in \mathbf{Z}$. If x and y have the same non-zero remainder on division by 3, then $3|(x + 2y)$.

4. Rigorously prove the following statement. You should expect to use "without loss of generality."

   Let $p, q, r \in \mathbf{Z}$. If exactly one of these is even, then $p + q + r$ is also even.