Math 235 - Dr. Miller - HW #3, Fall 2018 - **Due Friday, Sept. 14, 2018**

Write a rigorous direct proof of each statement below. In order to prove the informal statements, you'll have to rephrase them as formal conditionals.

Remember: correct proof structure always includes (1) assumptions, (2) NTS, (3) body, (4) conclusion, and (5) "exit move." Be sure to embed algebra/computations within sentences, not just as lists of equations like you might show on a calc or pre-calc problem. Finally, use FORMAL definitions when possible; childhood understanding of patterns is NOT acceptable "reasoning" in a proof.

1. If x, y, and z are all odd, then $3x + y - 5z$ is also odd.

2. The product of any two odd numbers is odd.

3. The sum of any three multiples of 11 is still a multiple of 11.

4. Let $a, b, c \in \mathbf{Z}$. If $a|b$ and $a|(2b + c)$, then $a|(3c)$.

5. *Linear Combination Theorem(+)*: Let $a, b, n, x, y \in \mathbf{Z}$. If $n|x$ and $n|y$, then $n|(ax+by)$.

6. If $x \in \{12n - 8 \mid n \in \mathbf{Z}\}$, then x is of the form $3m + 4$ for some integer m.

7. If $x, y \in \mathbf{Q}$, then $2x - y \in \mathbf{Q}$ also.

8. $\mathbf{Q}$ is closed under non-zero division.

9. Let $n \in \mathbf{Z}$. Then $3n^2 + 5n + 7$ is odd.

(+) Memorize the statement of this theorem. I may ask for it on Exam #1.