Math 235 - Dr. Miller - HW #3: Proof by Cases, "Or Conclusion," Ctp - Due by 4pm Friday, 9/13/2024

Work on your own paper, leaving plenty of room for my comments, and staple this page to the front.

Correct proof structure always includes (1) assumptions, (2) NTS, (3) body, (4) conclusion, and (5) "exit move." Embed algebra/computations within sentences, not just as lists of equations like you might show on a calc or pre-calc problem. Finally, use FORMAL definitions when possible; childhood understanding of patterns is NOT acceptable reasoning in a proof.

1. Rigorously prove the following statements directly. Use cases or "wlog" where appropriate.

   (a) If $c, d \in \mathbf{Z}$ have the same parity, then $c + d$ is even.

   (b) If x and y are integers, then $x - 2y$ has the same parity as x.

   (c) Let $x, y \in \mathbf{R}$. If $|x - 3| = 2$ and $|y + 1| = 0$, then $x + y$ is a multiple of 4. (You're doing BAD algebra if you don't encounter cases here.)

   (d) Let $n \in \mathbf{Z}$. If $3 \nmid n$, then n^2 has remainder 1 on division by 3.

   (e) Let $x, y \in \mathbf{Z}$. If they have different odd remainders on division by 5, then $5 \mid xy + 2$.

   (f) Let $x, y, z \in \mathbf{Z}$. If exactly two of them have the same parity, then $xy + yz + xz$ is also of that same parity.

   (g) Let $x, y, z \in \mathbf{Z}$. If at least one of them is divisible by 7, then xyz is too.

2. Complete a truth table (as in Discrete Math) to confirm the logical equivalences that govern each style of proof named below. Conclude each with a sentence describing what behavior of the table actually confirms equivalence.

   (a) Proof by cases: $(p \vee q) \rightarrow r \quad \equiv \quad (p \rightarrow r) \wedge (q \rightarrow r)$

   (b) "Or conclusion"-style proof: $p \rightarrow (q \vee r) \quad \equiv \quad (p \wedge \sim q) \rightarrow r$

3. Prove rigorously, using "or conclusion" style:

   (a) Prop. - Let $x, y \in \mathbf{R}$ with $y \neq 0$. If $\dfrac{3 - x}{y}$ is rational, then x is rational or y is irrational.

   (b) Prop. - Let $n \in \mathbf{Z}$. Prove that if $4 \nmid n$, then $n \div 4$ leaves remainder 3 or $4 | (n^2 + n + 2)$.

4. Prove rigorously by contrapositive:

   Prop. - Let $x, y \in \mathbf{Z}$. If xyz is even, then at least one of x, y, or z is even.