1. While truth tables are NOT a focus of this course, logical equivalence of statement forms legitimizes several proof styles that we do cover in Modern Concepts. In each task below, fill in the equivalent form that creates the named type of proof style, then make a truth table demonstrating that the two forms truly are logically equivalent:
   (a) $(p \vee q) \Rightarrow r \equiv$ ___?___ allows us to use proof by cases.
   (b) $p \Rightarrow (q \vee r) \equiv$ ___?___ allows us to use "or conclusion"-style proof.

2. Prove that if $x, y, z \in \mathbf{Z}$ all have the same remainder on division by 3, then $2x + 3y + 4z$ has remainder 0.

3. Prove that if n is a prime larger than 6, then $6|(n^2 - 1)$. (Hint: Division Algorithm!)

4. (a) Use "without loss of generality" appropriately in proving: If exactly two of $x, y, z \in \mathbf{Z}$ are odd, then $x + y + z$ is even.
   (b) Why would "without loss of generality" NOT be appropriate if the conclusion above had been that $3x + y - z$ is even?

5. Prove: if exactly one of $x, y, z, w \in \mathbf{Z}$ is odd, then $x + xy + xz + xw + y + w + z$ is odd.

6. Prove that if at least one of $x, y, z \in \mathbf{Z}$ is divisible by 5, then so is their product. (Careful! Try to avoid excessive "casing" here.)

7. Write an "or conclusion"-style proof of each statement below. Use careful algebra!
   (a) Let $x \in \mathbf{R}$. If $x^2 > 9$, then $x > 0$ or $x < -3$. (Yes, I mean zero vs. negative 3.)
   (b) Let $x, y \in \mathbf{R}$ with $y \neq 0$. If $\frac{2+x}{y}$ is rational, then x is rational or y is irrational.

8. Problem #11 on p.41 of our text is a "Proofs to Grade" problem. Such problems help you strengthen the ability to spot errors in your own proofs before I do! Work all parts of this problem, but you need **only turn in your answers to parts (b) and (e).** (The answers to (a), (c), and (d) are in the back of the book, so check your responses against those).