Math 235 - Dr. Miller - HW #4, Fall 2024 - **Due by 4pm Friday, Sept. 20, 2024**

1. For each proposition below, write a short sentence describing why proof by contrapositive (ctp) is a good choice for it, then rigorously prove, using that style of proof.

   (a) Prop. - Let a be a real number. If $(a+2)(a-3) < 0$, then $a < 3$.

   (You are ALLOWED to use algebra on inequalities $<$, $>$, $\leq$, $\geq$ here.)

   (b) Prop. - Circle C has center $(2,4)$. If $(0,3)$ is not inside C, then $(3,1)$ is also not inside C. *(Hint: negate carefully!*

   It's probably best here NOT to write equations of circles. Rather, use as the definition of a circle that it is the set of all points whose distance from the center is exactly the radius value. Points inside/outside the circle are at a distance that's less/greater than the radius. The distance formula should be useful.

2. Write a rigorous proof by contradiction for each proposition below.

   (a) Prop. - The square root of 20 is irrational. *(Don't imitate the $\sqrt{2}$ proof too closely. There's a small twist here.)*

   (b) Prop. - $\sqrt[5]{3}$ is irrational. *(And there's a different twist here.)*

   (c) Prop. - The graphs of $y = x^2 + 4x + 3$ and $y + 1 + (x-2)^2 = 0$ cannot intersect. *(Be careful of your function variables vs. proof variables.)*

   (d) Prop. - There is no largest even integer.

   (e) Prop. - Let $x, y \in \mathbf{R}$. If x is rational and y is irrational, then $x + y$ is irrational.

   (You may only use the definition of rational/irrational here. No lemmas about $\mathbf{Q}$ closure.)

3. Write a rigorous two-part proof of the following claims:

   (a) Proposition: Let $a, b, c \in \mathbf{Z}^+$. We have ac divides bc if and only if a divides b.

   (b) Proposition: Let y be an integer. Then $y^2 - 1$ is divisible by 3 if and only if y has remainder 1 or 2 on division by 3.