<u>Math 235 - Dr. Miller - REVISED HW #5, Fall 2018 - **Due Friday, Oct. 5, 2018**</u>

This assignment addresses proof by contrapositive and biconditional (2-directional) proof.

1. Determine which results below are best proved by contrapositive, rather than directly, and justify your choices in a sentence apiece. Do NOT actually prove.
   (a) Let $m, n \in \mathbf{Z}$. Then $m^2 + n^2$ is odd if m and n have different parity.
   (b) Let $m, n \in \mathbf{Z}$. Then $m^2 + n^2$ is odd only if m and n have different parity.
   (c) Let $a, b, c \in \mathbf{R}$. If $ab + c \notin \mathbf{Q}$, then at least one of $a, b, c \notin \mathbf{Q}$.
   (d) Let $p, q \in \mathbf{Z}^+$. If there exist $x, y \in \mathbf{Z}$ with $px + qy = 1$, then one of x or y is positive and the other negative.
   (e) Let $a, b, c \in \mathbf{Z}$. Then $a|b$ and $a|c$ only if $a|(b + c)$.
   (f) Let $a, b, c \in \mathbf{Z}$. Then if $a|b$ but $a \not| c$, $a \not| (b + c)$.

2. Choose TWO statements that you identified above as suited to proof by contrapositive, and prove them in that style.

3. Prove by contrapositive: Let $a, b \in \mathbf{Z}^+$. If $a < b$ and $ab < 3$, then $a = 1$. (Careful!)

4. Prove: Let $a, b, \in \mathbf{Z}^+$. Then $(a + 1)|b$ and $b|(b + 3)$ if and only if $a = 2$ and $b = 3$. (Hint: You can use as a lemma that for all integers $n \neq 0$, the only multiples of n that divide n are $-n$ and n itself.)

5. Prove: Let $c \in \mathbf{R}$. The graph of the function $y = x^2 + 2x + c$ intersects the x-axis if and only if $c \leq 1$. (Careful! This needs to be a formal, logically structured PROOF, not just computations or references to "facts" from precalculus that you likely didn't prove in that course.) Also, be careful about the distinction between the function variables x and y (which should NOT change) and proof variables such as c and others that YOU might need to introduce, like intercepts, etc.