Math 235 - Dr. Miller - HW #6, Fall 2018 - **Due Friday, Oct. 12, 2018**

1. Prove that the square root of 150 is irrational.

2. Prove that $\sqrt[7]{9}$ is irrational.

3. Prove that the graphs of $y = x^2 + 4x + 3$ and $y + 1 + (x - 2)^2 = 0$ cannot intersect.

   (As always, be careful not to mix-up the function **variables** x and y with letters that you might choose to represent actual real numbers that are **coordinates** of points.)

4. Prove (rigorously) that there is no smallest positive rational number.

5. Let $x, y \in \mathbf{Z}$. Consider this statement: if $x \neq y$, then the square of their sum cannot equal 4 times their product.
   (a) Write a proof by contradiction of the statement.
   (b) Now write a proof by contrapositive of the statement.
   (c) Informally describe the similarities and the differences in your two proofs.
   (d) Which one - based on previously assigned reading in our course - is better, stylistically, and why? (Look up Wikipedia's definition of "stylistics.")

6. (a) Let $x \in \mathbf{R}$. Prove by contradiction that if $x > 0$, then $x + \frac{1}{x} \geq 2$.
   (b) The statement in Problem #5 could be proved by both contradiction and contrapositive, but this statement isn't so lucky: when you try contrapositive-style here, there's a tricky spot. Describe exactly where that spot is - that is, where is the sentence in your #6a work where proof by contradiction lets us keep going, but proof by contrapositive gets stuck or forces a major change in logic?

7. Prove by any meaningful method: Every point on the line $y = 6 - x$ lies outside the circle with radius 4 and center $(-3, 1)$.

   (Hint: rewrite as a conditional or universal conditional first, then think about the proof style you want to use.)

8. As on earlier HW, assign grades to these text-book problems and briefly justify your choice: p. 50 #12(b)-(d) and (f).

   (I know the answer to part (b) is in the book, but try to answer on your own before looking.)