Math 235 - Dr. Miller - HW #7: Set Concepts and Proofs - **Due MONDAY, Oct. 21, 2024, by 5pm**

1. A reminder from Discrete Math: When we ask how many elements a set contains, we always mean *distinct* elements. The number of elements in a set is called its cardinality. Find the cardinality, denoted $n(S)$ or $|S|$, of each set below.

   (a) $T = \{t, o, o, n, t, n, y\}$

   (b) $F = \{x \mid x\ is\ a\ prime\ factor\ of\ 1430\}$

   (c) $P = \{x \mid x\ is\ a\ positive\ factor\ of\ 1430\}$

   (d) $E = \{\emptyset\}$

2. Deeper review of Discrete Math concepts for sets: Let $A = \{x \in \mathbf{Z}^+ \mid 2 < x \leq 8\}$ and $B = \{x \mid x\ is\ a\ prime\ number\ less\ than\ 7\}$.

   (a) Rewrite A and B using roster notation.

   (b) How many subsets does A have?

   (c) How many subsets does $A \times B$ have? Explain how you got your answer.

   (d) Are $\mathcal{P}(A \cup B)$ and $\mathcal{P}(A) \cup \mathcal{P}(B)$ equal? Justify your response informally.

   (e) Are $\mathcal{P}(A \cap B)$ and $\mathcal{P}(A) \cap \mathcal{P}(B)$ equal? Justify your response informally.

   (f) How many subsets does $\mathcal{P}(A \cup B)$ have? Explain how you got your answer.

3. Rigorously prove the following claims about "subsetness" and set equality.

   (a) Prop. - Let $T = \{8n - 7 \mid n \in \mathbf{Z}\}$. Then T is a subset of the set S of odd integers.

   (b) Prop. - Let R be the set of all numbers of the form $15k + 6$ where k is an integer, while S is the set of all numbers of the form $5n + 1$ where n is an integer. Then $R \subseteq S$ but $S \not\subseteq R$.

   (c) Let $S = \{\ldots, -5\pi/2, -3\pi/2, -\pi/2, \pi/2, 3\pi/2 \ldots\}$ and let $T = \{x \in \mathbf{R} \mid \sin(2x) = \cos(x)\}$. Then $S \subseteq T$. (Hint: Rewrite S in set-builder notation BEFORE beginning your proof, so that the statement "$s \in S$" has a definition.)

   (d) Prop. - $\mathbf{Z} = \{11x + 9y | x, y \in \mathbf{Z}\}$. (Hint: Make a universal hypothesis in which you give a letter name to the set on the right, such as "Let $A = \{11x + 9y | x, y \in \mathbf{Z}\}$.)

4. (a) Refer to the book's chain-style proof of Theorem 2.2.1(m). Rewrite that proof WITHOUT the incorrect parentheses, and labeling the end of each line with either the definition that was used or else the label "rules of logic."

   (b) Repeat this process for the book's proof of Theorem 2.2.3(a).

   (c) Write a similarly labeled, rigorous chain-style proof of: For all sets A, B, C, $(A \times B) \cap (C \times B) = (A \cap C) \times B$.

   (d) Also write a rigorous, labeled, chain-style proof of: For all sets G, H, $G \setminus (G \cap H) = G \setminus H$. (This one should require at least one "it's not the case that...")

5. Prove rigorously using any meaningful method (everything we've covered is fair game!):

   (a) Prop. - If A and B are any sets, then $A \cap B \neq \emptyset$ or $A \setminus B = A$.

   (b) Prop. - For all sets A, $A \times \emptyset = \emptyset$