1. Complete these definitional tasks from the textbook:

    (a) Problem #2 (a)-(c) on p. 161

    (b) Also Problem #2 (a)-(c) on p. 172

    (c) Problem #8 (a)-(c) on p.173, with brief explanation for each response

2. Rigorously prove the following:

    (a) Let $L = \{(a, b) \in \mathbf{R} \times \mathbf{R} \mid 3a + 4b = 12\}$. Then $\mathbf{R} = Dom(L)$.

    (b) Let F, G, and H be relations on a set A. If $F^{-1} \subseteq G$, then $G^{-1} \nsubseteq H$ or $F \subseteq H$.

3. Rigorously prove whether R in each part below is/is not reflexive, symmetric, and/or transitive. (So, yes, you are writing a proof about each of the three properties every time.)

    (a) On $\mathbf{Z}$, xRy if $5 \mid 4x + y$.

    (b) On $\mathbf{R}$, xRy if $x = y$ or $xy = 1$.

    (c) On $\mathcal{P}(\mathbf{Z})$, ARB if $2 \notin A \cup B$.

    (d) On $\mathbf{Z} \times \mathbf{Z}$, $(a, b)R(x, y)$ if $ax = -by$.

    Careful: We aren't relating an individual *element* x to another individual element y, but rather relating an entire *ordered pair* (a, b) to another one (x, y). Transitivity brings in a *third* pair.

4. Let $A = \{1, 2, 3, 4, 5\}$. For each part below, give a set B and an arrow diagram ($\neq$ digraph) for f with the specified qualities. You need not explain. The parts are separate, so answers will vary. Pay close attention to whether I am asking for B to be the domain or the codomain.

    (a) f is a relation from A to B, but not a function.

    (b) f is a one-to-one function from B to A, but not onto.

    (c) f is a function from A onto B.

5. For each relation below, identify its features by circling as instructed. No proof is required.

- Circle whether the relation **is** a function or **not**, with the given domain and codomain.
- For those that are functions, also circle whether it is onto/not AND one-to-one/not.

(a) $f : \mathbf{R} \longrightarrow \mathbf{R}$ via $f(x) = \sin x$

   Not a function   *Function that is : onto   not onto   1 − 1   not 1 − 1*

(b) $f : \mathbf{R} \longrightarrow [-1, 1]$ via $f(x) = \sin x$

   Not a function   *Function that is : onto   not onto   1 − 1   not 1 − 1*

(c) $f : \mathbf{R} \longrightarrow \mathbf{R}$ via $f(x) =$ an angle whose cosine equals x

   Not a function   *Function that is : onto   not onto   1 − 1   not 1 − 1*

(d) $f : \mathbf{R} \longrightarrow \mathbf{R}$ via $f(x) = e^x$

   Not a function   *Function that is : onto   not onto   1 − 1   not 1 − 1*

(e) $f : \mathbf{R} \longrightarrow \mathbf{R}$ via $f(x) =$ an exponent needed on e to create x as a result

   Not a function   *Function that is : onto   not onto   1 − 1   not 1 − 1*