

1. Let  $A, B$  be sets. (This is our universal hypothesis. Include it in your proofs below.)
  - (a) Prove that  $A \times B = \emptyset$  if and only if  $A = \emptyset$  or  $B = \emptyset$ .
  - (b) Is it true that  $A \cap B = \emptyset$  if and only if  $A = \emptyset$  or  $B = \emptyset$ ? If so, prove it; if not, disprove it.
  
2. Consider this statement: If  $A, B, C, D$  are sets with  $A \cup B \subseteq C \cap D$ , then  $A \cap B \subseteq C \cup D$ .
  - (a) Prove the statement directly, using only definitions of set operations and “subsetness.”
  - (b) Prove the statement directly again, this time using only these lemmas: Theorem 2.2.1(a) and (b), and Transitivity of Subsetness.
  - (c) Prove the statement by contrapositive, using your choice of ingredients (definitions vs. lemmas).
  
3. In a chain-style proof of an identity, we NEVER put parentheses in a proof just to group SENTENCES, yet p.97 of the text shows this bad format. RE-WRITE the proof of Theorem 2.2.1 (m) to use good English, not bad mergers of notation with words. Commas and semi-colons are your friends!
  
4. In a formal logic setting, we’re NEVER allowed to negate a statement just by slapping an “it is not the case/it is not true that” phrase in the front. However, in some set chain-style proofs, such a statement helps us in our goal to tease apart just one set operation at a time. Imitate the helpful use of “it is not the case that” from Theorem 2.2.2 (g)’s proof, to prove Theorem 2.2.2(h).
  
5. Prove via chain-style proof; follow the extra warnings in the problems above. Let  $A, B, C, D$  be sets as your universal hypothesis.
  - (a)  $(A - B) \times C = (A \times C) - (B \times C)$ .
  - (b)  $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$ .
  
6. Assign grades and justify your choice for Problem #20 (c)-(f) on p.103.