

1. (a) Prove that  $f : (0, 2] \rightarrow [1, \infty)$  via  $f(x) = \frac{2}{x}$  is a function.  
(b) Prove that  $f : \mathbf{R} \rightarrow \{-1, 1\}$  is NOT a function when

$$f(x) = \begin{cases} -1 & \text{if } x \text{ is negative} \\ 1 & \text{if } x \text{ is positive} \end{cases}$$

- (c) Prove that  $f : \mathbf{Z} \rightarrow \{2, 3, 0\}$  is NOT a function when

$$f(x) = \begin{cases} 2 & \text{if } 2|x \\ 3 & \text{if } 3|x \\ 0 & \text{if neither 2 nor 3 divides } x \end{cases}$$

2. [8 pts - 4 each] Prove that the following functions ARE onto:

- (a)  $f : \mathcal{P}(\{1, 2, 3, \dots, 100\}) \setminus \{\emptyset\} \rightarrow \{1, 2, 3, \dots, 100\}$  via  $f(X) =$  the largest member of  $X$   
(b)  $f : \mathbf{R} \rightarrow [-16, \infty)$  via  $f(x) = x^2 - 8x$

3. [4 pts - 2 each] Prove that the following functions are NOT onto:

- (a)  $f : \mathcal{P}(\{1, 2, 3, \dots, 100\}) \rightarrow \mathcal{P}(\{1, 2, 3, \dots, 100\})$  via  $f(X) = X \cup \{1, 2\}$   
(b)  $f : (0, 1] \rightarrow [1, \infty)$  via  $f(x) = \frac{3}{x}$

4. [4 pts - 2 each] Prove that the following functions are one-to-one:

- (a)  $f : (0, 2] \rightarrow (0, \infty)$  via  $f(x) = \frac{2}{\sqrt{x}}$   
(b)  $f : \mathbf{R}^+ \rightarrow \mathbf{R}$  via  $f(x) = 3x - 8 - \frac{1}{x}$

5. [4 pts - 2 each] Prove that the following functions are NOT one-to-one:

- (a)  $f : \mathcal{P}(\{1, 2, 3, \dots, 100\}) \rightarrow \mathcal{P}(\{1, 2, 3, \dots, 100\})$  via  $f(X) = X \cap \{1, 2\}$   
(b)  $f : \mathbf{R} \rightarrow [-16, \infty)$  via  $f(x) = x^2 - 8x$